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Assignment

Quantum Physics

PHYSICAL SCIENCES

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Assignment – 1: Fundamental of Quantum Mechanics

1. The de Broglie wavelength of particles of mass m with average momentum p at a temperature T in three dimensions is given by

(a) $\lambda = \frac{h}{\sqrt{2mk_BT}}$ (b) $\lambda = \frac{h}{\sqrt{3mk_BT}}$ (c) $\lambda = \frac{h}{\sqrt{2k_BT}}$ (d) $\lambda = \frac{h}{\sqrt{2m}}$

2. The angular frequency ω of deep water waves varies as the inverse square root of the wavelength λ, i.e., $\omega(\lambda) \propto 1/\sqrt{\lambda}$. Which of the following is the relation between its group velocity v_g and v_p?

(a) $v_g = v_p/2$ (b) $v_g = v_p$ (c) $v_g = 2v_p$ (d) $v_g = \lambda v_p$

3. The energy of a 200 nm photon is:
(a) 0.01 eV (b) 100 eV (c) 10 eV (d) 1 eV

4. Phase velocity and group velocity for a wave will be identical
(a) If the phase velocity is independent of wavelength.
(b) If the phase velocity is dependent on wavelength.
(c) If the group velocity is dependent on wavelength.
(d) If the group velocity is dependent on refractive index of the medium.

5. An electron in an X-ray tube is accelerated through a voltage V and strikes a tungsten target. The shortest wavelength photon emitted will have

(a) $\lambda = \frac{\hbar c}{eV}$ (b) $\lambda = \frac{eV}{hc}$ (c) $\lambda = \frac{hc}{eV}$ (d) $\lambda = \frac{eV}{\hbar c}$

6. The dispersion relation for a certain type of excitation is given by $\omega = Ak^a$, where A and a > 0 are constants. For these excitations, the group velocity is less than the phase velocity when

(a) a = 1 (b) $a = \sqrt{2}$ (c) a < 1 (d) $a \geq 2$

7. A small object is viewed with a microscope, using light of wavelength λ to illuminate the object. Let α be the half-angle subtended by the objective of the microscope at the position of the object being examined. The uncertainty Δx in the position of the object is of the order of

(a) $\lambda/\sin\alpha$ (b) $\lambda/\cos\alpha$ (d) $\lambda\sin\alpha$ (d) $2\lambda\sin\alpha$

8. The angular frequency ω of deep water waves varies as the inverse square root of the wavelength λ, i.e., $\omega(\lambda) \propto 1/\sqrt{\lambda}$. Which of the following is the relation between its group velocity v_g and v_p?

(a) $v_g = v_p/2$ (b) $v_g = v_p$ (c) $v_g = 2v_p$ (d) $v_g = \lambda v_p$

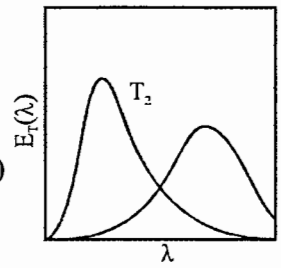
9. If a particle of mass $m = 5\times10^{-28}$ kg is confined to move in a space of size 10^{-15} m, the minimum value of its kinetic energy will approximately be

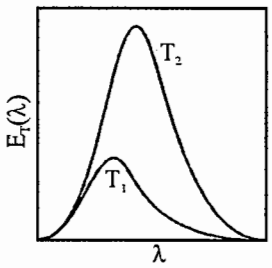
(a) 10^{-9} kg m^2/s^2 (b) 6.6×10^{-19} kg m^2/s^2
(c) 10^{-12} kg m^2/s^2 (d) 6.6×10^{-15} kg m^2/s^2

10. The velocity of an electron in the ground state of a hydrogen atom is v_H. If v_p be the velocity of an electron in the ground state of postironium, then

(a) $v_p = v_H$ (b) $v_p = 2v_H$ (c) $v_p = \frac{v_H}{2}$ (d) $v_p = \sqrt{2}v_H$

11. Which of the graphs below gives the correct qualitative behavior of the energy density $E_T(\lambda)$ of blackbody radiation of wavelength λ at two temperatures T_1 and T_2 ($T_1 < T_2$)?

(a)


(b)


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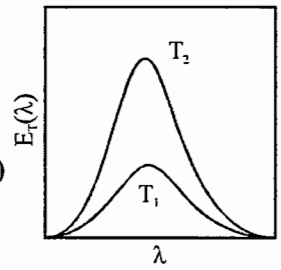
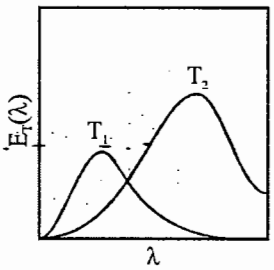


(d)


[ANSWERS]

1. (b) 2. (a) 3. ( ) 4. (a) 5. (c) 6. (c) 7. (a)
8. (a) 9. ( ) 10. ( ) 11. (c)

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Assignment – 2: Operators

1. $\hat{A}$ and $\hat{B}$ represents two physical characteristics of a quantum system. If $\hat{A}$ is Hermitian, then for the product $\hat{A}\hat{B}$ to be Hermitian, it is sufficient that
   (a) $\hat{B}$ is Hermitian
   (b) $\hat{B}$ is anti-Hermitian
   (c) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ commute
   (d) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ is anti-commute.

2. If $[x,p]=i\hbar$, the value of $[x^3,p]$ is :
   (a) $2i\hbar x^2$ (b) $-2i\hbar x^2$ (c) $3i\hbar x^2$ (d) $-3i\hbar x^2$

3. The commutator $[x,P^2]$, where x and P are position and momentum operators respectively, is
   (a) $2i\hbar P$ (b) $-i\hbar P$ (c) $2i\,\hbar\, x\, P$ (d) $-2i\,\hbar\, x\, P$

4. The commutator $[x,p_x^2]$ of the quantum mechanical operators x and p_x^2 is given by
   (a) $2i\hbar$ (b) $2i\hbar p_x$ (c) $-2i\hbar p_r$ (d) $2i\hbar x$

5. Two Hermitian operators A and B commute (on a finite dimensional linear vector space). This implies that
   (a) every eigenvector of A is necessarily an eigenvector of B.
   (b) a complete set of simultaneous eigenvector of A and B exist.
   (c) every vector is necessarily an eigenvector of A and B.
   (d) all eigenvectors of A are always orthogonal to those of B.

6. An electron moving in one dimension $(-\infty < x < \infty)$ in a potential of period L is in eigenstate of its Hamiltonian. Its wavefunction is of the form $u_k(x)\exp(ikx)$. The $u_k(x+L)$ is equal to
   (a) $Lu_k(x)$ (b) $u_k(x-L)$ (c) $(1/L)u_k(x)$ (d) $u_k(x)$

7. Let A and B be the Hermitian operators correspoinding to two physical observables of a system, such that $[A,B]\neq 0$. We may conclude in general that
   (a) The uncertainty product is always $(\Delta A)(\Delta B) > \frac{h}{2}$
   (b) The system can never be in a state in which $(\Delta A)(\Delta B)=0$
   (c) Neither A nor B can have any eigenstates.
   (d) A and B may have one or more common eigenstate(s), but not a complete set of these.

8. Let $N = b^\dagger b$, where the operators b and $b^\dagger$. satisfy the relations $bb^\dagger + b^\dagger b = 1$ and $b^2 = 0 = (b^\dagger)^2$. The eigenvalues of N are
   (a) Positive and negative integers
   (b) all positive integers
   (c) ± 1 and 0 only
   (d) 0 and 1 only

9. The commutator $[\hat{x},[\hat{x},\hat{p}]]$ is
   (a) $i\hbar\hat{x}$ (b) $-i\hbar\hat{x}$ (c) $2i\hbar\hat{x}$ (d) 0

10. An operator $\hat{F}$ is such that $\hat{F}\psi(x)=\psi(x+a)$. Here $\psi(x)$ is any wave function in the position representation and a is a constant. If $\hat{x}$ and $\hat{p}$ are position and momentum operators, then $\hat{F}$ is given by

(a) $\exp\left(\frac{-ia\hat{p}}{\hbar}\right)$ (b) $\exp\left(\frac{ia\hat{x}}{\hbar}\right)$ (c) $\exp\left(\frac{-ia\hat{x}}{\hbar}\right)$ (d) $\exp\left(\frac{ia\hat{p}}{\hbar}\right)$

11. If the momentum operator $\hat{p}$ is represented by multiplication by p on a wave function of a particle in one dimension, the position operator $\hat{x}$ is given by:

(a) $\hat{x}=i\hbar\frac{\partial}{\partial p}$ (b) $\hat{x}=-i\hbar\frac{\partial}{\partial p}$ (c) $\hat{x}=\frac{\partial}{\partial p}$ (d) $\hat{x}=x$ (multiplication by x)

12. Three operators X, Y and Z satisfy the commutation relations

$$[X,Y]=i\hbar Z,[Y,Z]=i\hbar X \text{ and } [Z,X]=i\hbar Y$$

The set of all possible eigenvalues of the operator Z, in units of $\hbar$, is

(a) $\{0,\pm1,\pm2,\pm3,.....\}$ (b) $\left\{\frac{1}{2},1,\frac{3}{2},2,\frac{5}{2},.....\right\}$

(c) $\left\{0,\pm\frac{1}{2},\pm1,\pm\frac{3}{2},\pm2,\pm\frac{3}{2},.....\right\}$ (d) $\left\{-\frac{1}{2},+\frac{1}{2}\right\}$

13. $\hat{A}$ and $\hat{B}$ are two quantum mechanical operators. If $\left[\hat{A},\hat{B}\right]$ stands for the commutator of $\hat{A}$ and $\hat{B}$, then $\left[\left[\hat{A},\hat{B}\right],\left[\hat{B},\hat{A}\right]\right]$ is equal to

(a) $\hat{A}\hat{B}\hat{A}\hat{B}-\hat{B}\hat{A}\hat{B}\hat{A}$ (b) $\hat{A}\left(\hat{A}\hat{B}-\hat{B}\hat{A}\right)-\hat{B}\left(\hat{B}\hat{A}-\hat{A}\hat{B}\right)$ (c) zero (d) $\left(\left[\hat{A},\hat{B}\right]\right)^2$

14. x and p are two operators which satisfy $c[x,p]=i$. the operators X and P are defined as $X=x\cos\phi+p\sin\phi$ and $Y=-x\sin\phi+p\cos\phi$ for ϕ real. Then [X, Y] equals

(a) 1 (b) -1 (c) i (d) $-i$

15. For any operator A, $i\left(A^{+}-A\right)$ is:

(a) Hermitian (b) Anti-Hermitian (c) Unitary (d) Orthogonal

16. The Hamiltonian of a three-level system is:

$$H=a\begin{pmatrix}2&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

The energy eigenvalues are

(a) 2a, a, $-a$ (b) 2a, a, 0 (c) 2a, 0, 0 (d) 2a, a, a

17. The operators x and p_x are represented as $p_x\to\hat{p}_x, x\to i\hbar\frac{\partial}{\partial p_x}$. The value of $\left[\hat{x},\hat{p}_x\right]$ is:

(a) zero (b) $-i\hbar c$ (c) $i\hbar$ (d) $\hbar$

18. Define $\sigma_x=\left(f^{\dagger}+f\right)$, and $\sigma_y=-i\left(f^{\dagger}-f\right)$, where the σ's are Pauli spin matrices and $f, f^{\dagger}$ obey anticommutations relations $\{f,f\}=0\{f,f^{\dagger}\}=1$. then σ_z is given by

(a) $f^{\dagger}f-1$ (b) $2f^{\dagger}f-1$ (c) $2f^{\dagger}f+1$ (d) $f^{\dagger}f$

19. The commutator $\left[x^2, p^2\right]$ is:

(a) $2\,i\hbar\,xp$ (b) $2i\hbar(xp+px)$ (c) $2i\hbar px$ (d) $2i\hbar(xp-px)$

20. Which of the following is a self-adjoint operator in the spherical polar coordinate system (r,θ,φ)?

(a) $-\frac{i\hbar}{\sin^2\theta}\frac{\partial}{\partial\theta}$ (b) $-i\hbar\frac{\partial}{\partial\theta}$ (c) $-\frac{i\hbar}{\sin\theta}\frac{\partial}{\partial\theta}$ (d) $-i\hbar\sin\theta\frac{\partial}{\partial\theta}$

21. Given the usual canonical commutation relations, the commutator $[A,B]$ of $A=i(xp_y-yp_x)$ and $B=(yp_z+zp_y)$ is:

(a) $\hbar(xp_z-p_xz)$ (b) $-\hbar(xp_z-p_xz)$ (c) $\hbar(xp_z+p_xz)$ (d) $-\hbar(xp_z+p_xz)$

22. A proton is confined to a cubic box, whose sides have length 10^{-12} *m*. What is the minimum kinetic energy of the proton? The mass of proton is 1.67×10^{-27} *kg* and Planck's constant is 6.63×10^{-34} *Js*.
(a) $1.1\times10^{-17}\,J$ (b) $3.3\times10^{-17}\,J$ (c) $9.9\times10^{-17}\,J$ (d) $6.6\times10^{-17}\,J$

23. The operator $\left(\frac{d}{dx}-x\right)\left(\frac{d}{dx}+x\right)$ is equivalent to

(a) $\frac{d^2}{dx^2}-x^2$ (b) $\frac{d^2}{dx^2}-x^2+1$

(c) $\frac{d^2}{dx^2}-x\frac{d}{dx}x^2+1$ (d) $\frac{d^2}{dx^2}-2x\frac{d}{dx}-x^2$

24. The hermitian conjugate of the operator $(-\partial/\partial x)$ is
(a) $\partial/\partial x$ (b) $-\partial/\partial x$ (c) $i\partial/\partial x$ (d) $-i\partial/\partial x$

25. If the operators A and B satisfy the commutation relation [A, B] = I, where I is the identity operator, then
(1) $[e^A, B] = e^A$ (2) $[e^A, B] = [e^B, A]$
(3) $[e^A, B] = [e^{-B}, A]$ (4) $[e^A, B] = 1$

26. If A, B and C are non-zero Hermitian operators, which of the following relations must be false?
(a) [A, B] = C (b) AB + BA = C (c) ABA = C (d) A+ B = C

[ANSWERS]

1. (c)	**2. (c)**	**3. (a)**	**4. (b)**	**5. (b)**	**6. (d)**	**7. (b)**
8. (d)	**9. (d)**	**10. (d)**	**11. (a)**	**12. (c)**	**13. (c)**	**14. (c)**
15. (a)	**16. (a)**	**17. (c)**	**18. (b)**	**19. (b)**	**20. (c)**	**21. (c)**
22. (c)	**23. ( )**	**24. ( )**	**25. ( )**	**26. (a)**		

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Assignment – 3: Schrodinger

1. A particle of mass m is represented by the wavefunction $\psi(x) = Ae^{ikx}$, where k is the wavevector and A is a constant. The magnitude of the probability current density of the particle is

(a) $|A|^2 \frac{\hbar k}{m}$ (b) $|A|^2 \frac{\hbar k}{2m}$ (c) $|A|^2 \frac{(\hbar k)^2}{m}$ (d) $|A|^2 \frac{(\hbar k)^2}{2m}$

2. If the probability that x lies between x and x+dx is $p(x)dx = ae^{-ax}$, where $0 < x < \infty$, $a > 0$, then the probability that x lies between x_1 and x_2 $(x_2 > x_1)$ is:

(a) $\left(e^{-ax_1} - e^{-ax_2}\right)$ (b) $a\left(e^{-ax_1} - e^{-ax_2}\right)$ (c) $e^{-ax_2}\left(e^{-ax_1} - e^{-ax_2}\right)$ (d) $e^{-ax_1}\left(e^{-ax_1} - e^{-ax_2}\right)$

3. A system in a normalized state $|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle$, with $|\alpha_1\rangle$ and $|\alpha_2\rangle$ representing two different eigenstates of the system, requires that the constants c_1 and c_2 must satisfy the condition

(a) $|c_1|.|c_2| = 1$ (b) $|c_1| + |c_2| = 1$ (c) $\left(|c_1| + |c_2|\right)^2 = 1$ (d) $|c_1|^2 + |c_2|^2 = 1$

4. A normalized state of a particle moving in a potential $V(x)$ is given by

$$\psi(x,t) = \sum_{n=0}^{\infty} C_n \phi_n(x) e^{-F_n t/7}$$

where $\phi_n(x)$'s are the normalized eigenfunctions of the particle corresponding to the energies E_n's. Then

(a) $\sum_{n=0} |C_n|^2 = 1$

(b) The average energy of the particle in the state $\psi(x,t)$ is $\sum_{n=\infty}^{\infty} |C_n| E_n$

(c) $\psi(x,t)$ is an eigenfunction of the Hamiltonian of the particle.

(d) $\psi(x,t)$ is an eigenfunction of the momentum operator.

5. Which of the following functions represents acceptable wave function of a particle in the range $-\infty \le x \le \infty$.

(a) $\varphi(x) = A \tan x, A > 0$ (b) $\varphi(x) = B \cos x$, B real

(c) $\varphi(x) = C \exp\left(-D/x^2\right), C > 0, D < 0$ (d) $\varphi(x) = E\, x \exp\left(-Fx^2\right), E, F > 0$

6. Probability currentdensity in quantum mechanics is represented by the operator.

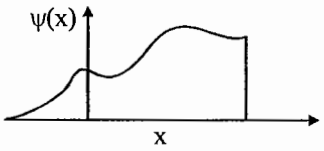
(a) $\frac{i\hbar}{2m}\left(\psi\bar{\nabla}\psi^* - \psi^*\bar{\nabla}\psi\right)$ (b) $\frac{\hbar}{2m}\left(\psi\bar{\nabla}\psi^* - \psi^*\bar{\nabla}\psi\right)$

(c) $\frac{\hbar}{2m}\left(\psi\bar{\nabla}\psi^* + \psi^*\bar{\nabla}\psi\right)$ (d) $\frac{i\hbar}{2m}\left(\psi\bar{\nabla}\psi^* + \psi^*\bar{\nabla}\psi\right)$

7. Let the hermitian operator H be the Hamiltonian of a quantum mechanical particle, E_0 its ground state energy, and $\psi(\vec{r})$ an arbitrary normalized trial wavefunction. The quantity $\int \psi^*(\vec{r}) H \psi(\vec{r}) dV$
(a) must necessarily be less than or equal to E_0.
(b) must necessarily be greater than or equal to E_0.
(c) may be any real number — greater than, equal to, or less than E_0.
(d) is purely imaginary.

8. The wave function $\psi(\vec{r})$ of a particle moving in three-dimensional space has the physical dimensions of
(a) $(\text{Length})^{-3/2}$ (b) $(\text{Length})^{3/2}$ (c) $(\text{Length})^{-1}$ (d) (Length)

9. The plot of a particular function is as shown



(a) This is not acceptable as a bound state wave function because it is not zero at the origin.
(b) This is not acceptable as a bound state wave function because it is discontinuous.
(c) This is acceptable as a bound state wave function.
(d) This is not acceptable as a bound state wave function but acceptable as a scattering state wave function.

10. Let $\psi = c_1\phi_1 + c_2\phi_2 + c_3\phi_3$, ϕ_1, ϕ_2, ϕ_3 being (orthogonal) eigen functions of H then,
(a) ψ is always an eigenfunction of H. (b) ψ can never be an eigen function of H
(c) ψ is an eigen function only if ϕ_1, ϕ_2, ϕ_3 are degenerate.
(d) ψ is an eigen function if any two of ϕ_1, ϕ_2, ϕ_3 are degenerate.

11. A is an operator which does not vary with time, and $\langle A \rangle$ the expection value of A in the state $|\psi\rangle$. Which of the following equation is correct in the Schrodinger representation.
(a) $\frac{\partial}{\partial t}\langle A \rangle = \frac{1}{i\hbar}\langle [A, H] \rangle$ (b) $\frac{d}{dt}\langle A \rangle = +\frac{1}{i\hbar}\langle [A, H] \rangle$
(c) $\frac{\partial}{\partial t}\langle A \rangle = -\frac{1}{i\hbar}\left\langle \frac{1}{H - A} \right\rangle$ (d) $\frac{d}{dt}\langle A \rangle = 0$

12. The Schordinger equation for a quantum system is a linear differential equation of the type
(a) First order in time and first order in space
(b) First order in space and second order in time.
(c) Second order in space and first order in time.
(d) Second order in space and second order in time.

13. The wave function of a particle at time t = 0 is given by $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|u_1\rangle + |u_2\rangle)$, where $|u_1\rangle$ and $|u_2\rangle$ are the normalized eigenstates with eigenvalues E_1 and E_2 respectively, $(E_2 > E_1)$. The shortest time after which $|\psi(t)\rangle$ will become orthogonal to $|\psi(0)\rangle$ is

(a) $\frac{-\hbar\pi}{2(E_2 - E_1)}$ (b) $\frac{\hbar\pi}{E_2 - E_1}$ (c) $\frac{\sqrt{2}\,\hbar\pi}{E_2 - E_1}$ (d) $\frac{2\hbar\pi}{E_2 - E_1}$

Common data for Q. 14 and Q. 15

The wavefunction of a particle moving in free space is given by, $\psi = e^{ikx} + 2e^{-ikx}$

14. The energy of the particle is

(a) $\frac{5\hbar^2k^2}{2m}$ (b) $\frac{3\hbar^2k^2}{4m}$ (c) $\frac{\hbar^2k^2}{2m}$ (d) $\frac{\hbar^2k^2}{m}$

15. The probability current density for the real part of the wavefunction is

(a) 1 (b) $\frac{\hbar k}{m}$ (c) $\frac{\hbar k}{2m}$ (d) 0

Common data for Q.16 and Q.17.

A free particle of mass m moves along the x direction. At t = 0, the normalized wave function of the particle is given by $\psi(x,0) = \frac{1}{(2\pi\alpha)^{1/4}} \exp\left[-\frac{x^2}{4\alpha^2} + ix\right]$, where α is a real constant.

16. The expectation value of the momentum, in this state is:

(a) $\hbar\alpha$ (b) $\hbar\sqrt{\alpha}$ (c) α (d) $\hbar/\sqrt{\alpha}$

17. The expectation value of the particle energy is:

(a) $\frac{\hbar^2}{2m}\frac{1}{2\alpha^{1/2}}$ (b) $\frac{\hbar^2}{2m}\alpha^2$ (c) $\frac{\hbar^2}{2m}\frac{4\alpha^2+1}{4\alpha^{1/2}}$ (d) $\frac{\hbar^2}{8m\alpha^{1/2}}$

18. A particle is in the normalized state $|\psi\rangle$ which is superposition of the energy eigenstates $|E_0 = 10\,eV\rangle$ and $|E_1 = 30\,eV\rangle$. The average value of energy of the particle in the state $|\psi\rangle$ is 20 eV. The state $|\psi\rangle$ is given by

(a) $\frac{1}{2}|E_0 = 10\,eV\rangle + \frac{\sqrt{3}}{4}|E_1 = 30\,eV\rangle$ (b) $\frac{1}{\sqrt{3}}|E_0 = 10\,eV\rangle + \sqrt{\frac{2}{3}}|E_1 = 30\,eV\rangle$

(c) $\frac{1}{2}|E_0 = 10\,eV\rangle - \frac{\sqrt{3}}{4}|E_1 = 30\,eV\rangle$ (d) $\frac{1}{\sqrt{2}}|E_0 = 10\,eV\rangle - \frac{1}{\sqrt{2}}|E_1 = 30\,eV\rangle$

19. The wave function of a particle, moving in a one-dimensional time-independent potential V(x), is given by $\psi(x) = e^{-iax+b}$, where a and b are constants. This means that the potential $V(x)$ is of the form

(a) $V(x) \propto x$ (b) $V(x) \propto x^2$ (c) $V(x) = 0$ (d) $V(x) \propto e^{-ax}$

20. The wavefunction of a particle in a one-dimensional potential at time $t = 0$ is

$$\psi(x, t=0) = \frac{1}{\sqrt{15}}\left[2\psi_0(x) - \psi_1(x)\right]$$

where $\psi_0(x)$ and $\psi_1(x)$ are the ground and the first excited states of the particle with corresponding energies E_0 and E_1. The wavefunction of the particle at a time 't' is:

(a) $\frac{1}{\sqrt{5}} e^{i(E_0+E_1)t/2\hbar}\left[2\psi_0(x) - \psi_1(x)\right]$ (b) $\frac{1}{\sqrt{5}} e^{-iE_0t/\hbar}\left[2\psi_0(x) - \psi_1(x)\right]$

(c) $\frac{1}{\sqrt{5}} e^{-iE_0t/\hbar}\left[2\psi_0(x) - \psi_1(x)\right]$ (d) $\frac{1}{\sqrt{15}}\left[2\psi_0(x)e^{-iE_0t/\hbar} - \psi_1(x)e^{-iE_1t/\hbar}\right]$

21. Which of the following is an allowed wavefunction for a particle in a bound state? N is a constant and $\alpha, \beta > 0$

(a) $\psi = N\frac{e^{-\alpha r}}{r^3}$ (b) $\psi = N\left(1-e^{-\alpha r}\right)$

(c) $\psi = Ne^{-\alpha x}e^{-\beta\left(x^2+y^2+z^2\right)}$ (d) $\psi = \begin{cases} \text{non-zero constant} & \text{if } r < R \\ 0 & \text{if } r > R \end{cases}$

22. Which of the following wave function is acceptable as the solution of the Schrodinger equation for all values of x?

(a) $\psi(x) = A\sec x$ (b) $\psi(x) = A\tan x$ (c) $\psi(x) = Ae^{x^2}$ (d) $\psi(x) = Ae^{-x^2}$

23. Which of the following wavefunctions can be bound-state solution of the one-dimensional Schrodinger equation for all values of x?

(a) $\psi = A\sec x$ (b) $\psi = A\,e^{x}$ (c) $\psi = A\,e^{x^2}$ (d) $\psi = A\,e^{-x^2}$

24. The ground state (apart form normalization) of a particle of unit mass moving in a one-dimensional potential $V(x)$ is $\exp\left(-x^2/2\right)\cosh\left(\sqrt{2}.x\right)$. The potential $V(x)$, in suitable units so that $\hbar = 1$, is (up to an additive constant).

(a) $x^2/2$ (b) $x^2/2 - \sqrt{2}\,x\tan h\left(\sqrt{2}\,x\right)$

(c) $x^2/2 - \sqrt{2}\,x\tan\left(\sqrt{2x}\right)$ (d) $x^2/2 - \sqrt{2}\,x\cot h\left(\sqrt{2}\,x\right)$

25. If a particle is represented by the normalized wave function

$$\psi(x) = \begin{cases} \dfrac{\sqrt{15}\left(a^2-x^2\right)}{4a^{5/2}} & \text{for } -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

the uncertainty Δp in its momentum is

(a) $\frac{2\hbar}{5a}$ (b) $\frac{5\hbar}{2a}$ (c) $\frac{\sqrt{10}\,\hbar}{a}$ (d) $\frac{\sqrt{5}\,\hbar}{\sqrt{2}\,a}$

26. Consider the wave function $Ae^{ikr}(r_0/r)$, where A is the normalization constant. For $r = 2r_0$, the magnitude of probability current density up to two decimal places, in units of $(A^2\hbar k/m)$, is _____.

27. If the expectation value of the momentum is $\langle p\rangle$ for the wavefunction $\psi(x)$, then the expectation value of momentum for the wavefunction $e^{ikx/\hbar}\psi(x)$ is

(a) k (b) $\langle p\rangle - k$ (c) $\langle p\rangle + k$ (d) $\langle p\rangle$

Assignment -3: Schrodinger

[ANSWERS]

1. (a)	2. (a)	3. (d)	4. (a,c)	5. (d)	6. (a)	7. (b)
8. (a)	9. (c)	10. (c)	11. (a)	12. (c)	13. (b)	14. (c)
15. (d)	16. (b)	17. (c)	18. (d)	19. (c)	20. (d)	21. (c)
22. (d)	23. (d)	24. (b)	25. (d)	26. (0.25)	27. ( )	

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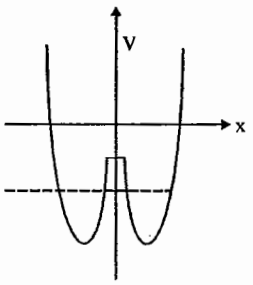
Assignment – 4: One-dimensional Potential

1. A particle with energy E is in a time-independent double well potential as shown in the figure. Which of the following statements about the particle is NOT correct?



(a) The particle will always be in a bound state.
(b) The probability of finding the particle in one well will be time-dependent.
(c) The particle will be confined to any one of the wells.
(d) The particle can tunnel from one well to the other, and back.

2. A one-dimensional harmonic oscillator is in the state

$$\psi(x)=\frac{1}{\sqrt{14}}\left[3\psi_0(x)-2\psi_1(x)+\psi_2(x)\right]$$

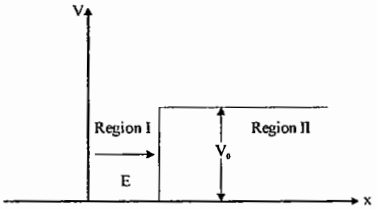
Where $\psi_0(x), \psi_1(x)$ and $\psi_2(x)$ are the ground, first excited and second excited states, respectively. The probability of finding the oscillator in the ground state is:

(a) 0 (b) $\frac{3}{\sqrt{14}}$ (c) $\frac{9}{14}$ (d) 1

3. A free particle is moving in +x direction with a linear momentum p. The wavefunction of the particle normalised in a length L is:

(a) $\frac{1}{\sqrt{L}}\sin\frac{p}{h}x$ (b) $\frac{1}{\sqrt{L}}\cos\frac{p}{h}x$ (c) $\frac{1}{\sqrt{L}}e^{-i\frac{p}{h}x}$ (d) $\frac{1}{\sqrt{L}}e^{i\frac{p}{h}x}$

4. A partricle is incident with a constant energy E on a one-dimensional potential barrier as shown in the figure.



The wavefunctions in regions I and II are respectively
(a) decaying, oscillatory
(b) oscillatory, oscillatroy
(c) oscillatory, decaying
(d) decaying, decaying.

5. A particle of energy E is incident on a rectangular potential barrier of height V_0 and width b.
(a) The reflection coefficient for $E<V_0$ tends to unity as b tends to infinity.
(b) The reflection coefficient is zero for some specific values of $E > V_0$.
(c) The transmission coefficient is always zero for $E < V_0$.
(d) The transmission coefficient is zero for some specific values of $E>V_0$.

6. In the first excited state of a one-dimensional harmoic oscillator with angular frequency ω, the energy eigenvalue is given by

(a) $\frac{1}{2}\hbar\omega$ (b) $\hbar\omega$ (c) $\frac{3}{2}\hbar\omega$ (d) $2\hbar\omega$

7. For the ground state of a particle moving freely in a one-dimensional box $0 \le x \le L$ with rigid reflecting end-points, the uncertainty product $(\Delta x)(\Delta p)$ is:

(a) h/2 (b) $\sqrt{2}$ h (c) > h/2 (d) $h/\sqrt{3}$

8. The ground state energy of a particle in an infinite square-well potential of width L is E. If the width of the wall is reduced to L/2, then the ground state energy becomes
(a) 2E (b) E/2 (c) 4E (d) E/4

9. Consider the operators a and a^+, satisfying the commutation relation $\left[a, a^+\right] = I$, the unit operator. There are no normalisable eigenstates of the operator
(a) a^+a (b) a (c) a^+ (d) $\left(a^+a\right)^2$

10. The first excited state of a one dimensional harmonic oscillator has eigenfunction $\phi(x) = Nxe^{-\alpha^2x^2/2}$ where N is:
(a) $\sqrt{2\alpha^3/\sqrt{\pi}}$ (b) $2\alpha^3/\sqrt{\pi}$ (c) $\sqrt{\alpha^2/\pi}$ (d) α/π

11. The expectation value of the momentum in the ground state of one - dimensional harmonic oscillator of mass m and frequency ω is:
(a) $\frac{1}{\sqrt{2m\hbar\omega}}$ (b) $\sqrt{2m\hbar\omega}$ (c) 0 (d) ∞

12. The unnormalized ground state wave function of a partial confined to a one dimensional box of length L is:
(a) $A\sin\left(\frac{2\pi x}{L}\right)$ (b) $A\sin\left(\frac{\pi x}{L}\right)$ (c) $A\sin\left(\frac{L}{\pi x}\right)$ (d) $A\sin\left(\frac{L}{2\pi x}\right)$

13. A stream of particles of mass m and energy E encounter a potential step of height W (<E). In the limit W approaching E, the fraction of particles reflected from the barrrier will
(a) Appraoch zero (b) Appraoch unity (c) Diverge to infinity (d) Approach half.

14. Consider one dimensional attractive potential of arbitrary shape. Which of the following statement is true?
(a) Only one bound state exists.
(b) It has at least one bound state.
(c) It may have bound state only if the depth of the potential is greater than some critical depth.
(d) It has no bound state.

15. For a one-dimensional harmonic oscillator, let $\psi_n(x)$ denote the eigenstate of the n^{th} quantum state. Which one of the statements is correct?
(a) Parity of $\psi_n(x)$ is even when n is even. (b) Parity of $\psi_n(x)$ is odd when n is even.
(c) Parity of $\psi_n(x)$ is even where n is odd (d) $\psi_n(x)$ has mixed parity.

16. The wavefunction of a particle is given by $\psi = \left(\frac{1}{\sqrt{2}}\phi_0 + i\phi_1\right)$, where ϕ_0 and ϕ_1 are the normalized eigenfnctions with energies E_0 and E_1 corresponding to the ground state and first excited state, respectively. The expectation value of the Hamiltonian in the state ψ is:
(a) $\frac{E_0}{2} + E_1$ (b) $\frac{E_0}{2} - E_1$ (c) $\frac{E_0 - 2E_1}{3}$ (d) $\frac{E_0 + 2E_1}{3}$

17. A particle is confined to the region $x \geq 0$ by a potential which increases linearly as $u(x) = u_0x$. The mean position of the particle at temperature T is:
(a) $\frac{k_BT}{u_0}$ (b) $(k_BT)^2/u_0$ (c) $\sqrt{\frac{k_BT}{u_0}}$ (d) u_0k_BT

18. A Particle in a one-dimensional potential has the wavefunction

$$\psi(x)=\frac{1}{\sqrt{a}}\exp\left(\frac{-|x|}{a}\right)$$

Where 'a' is a constant. It follows that for a positive constant V_0, the potential $V(x)=$

(a) V_0x^2 (b) $V_0|x|$ (c) $-V_0\delta(x)$ (d) $-V_0/|x|$

19. Consider a praticle in a one dimensional potential that satisfies $V(x)=V(-x)$. Let $|\psi_0\rangle$ and $|\psi_1\rangle$ denote the ground and the first excited states, respectively, and let $|\psi\rangle=\alpha_0|\psi_0\rangle+\alpha_1|\psi_1\rangle$ be a normalized state with α_0 and α_1 being real constants. The expectation value $\langle x\rangle$ of the position operator x in the state $\langle\psi|$ is given by

(a) $\alpha_0^2\langle\psi_0|x|\psi_0\rangle+\alpha_1^2\langle\psi_1|x|\psi_1\rangle$ (b) $\alpha_0\alpha_1\left[\langle\psi_0|x|\psi_1\rangle+\langle\psi_1|x|\psi_0\rangle\right]$

(c) $\alpha_0^2+\alpha_1^2$ (d) $2\alpha_0\alpha_1$

20. Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground and first excited state of a one dimensional harmonic oscillator. The uncertainty Δp in the state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$, is:

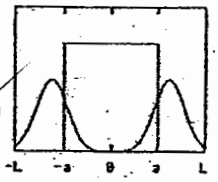
(a) $\Delta p=\sqrt{\hbar m\omega}/2$ (b) $\Delta p=\sqrt{\hbar m\omega/2}$

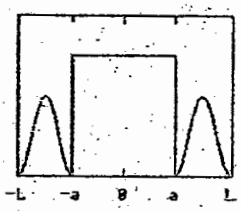
(c) $\Delta p=\sqrt{\hbar m\omega}$ (d) $\Delta p=\sqrt{2\hbar m\omega}$

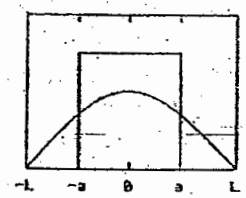
21. The schematic form of the ground state wave-function in the one-dimensional potential.

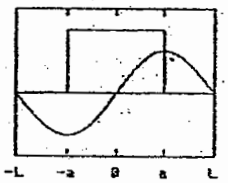
$$V(x)=\begin{cases}V_0, & \text{for } a\geq|x|\\ 0, & \text{for } a<|x|<L\\ \infty, & \text{for } |x|\geq L\end{cases}$$

where V_0, a positive constant, is given by

(a) 

(b) 

(c) 

(d) 

22. A particle of mass m in one dimension moves in a potential $V(x)=\frac{k}{2a}\left(1-e^{-ax^2}\right)$, where k and α are positive constants. If the particle executes small oscillations round x= 0, its quantum mechanical zero-point energy is

(a) $\frac{1}{2}\sqrt{\frac{\hbar^2 k}{m}}$ (b) $\frac{1}{2}\sqrt{\frac{\hbar^2 k}{m\alpha}}$ (c) $\frac{1}{2}\sqrt{\frac{\hbar^2 m}{k}}$ (d) $\frac{1}{2}\sqrt{\frac{\hbar^2 m\alpha}{k}}$

23. A particle is confined to a one dimensional harmonic oscillator potential in the region $0 < x < \infty$. At x = 0 there is n infinite barrier. The energy levels of the particle are separated by

(a) $\frac{1}{2}\hbar\omega$ (b) $\hbar\omega$ (c) $\frac{3}{2}\hbar\omega$ (d) $2\hbar\omega$

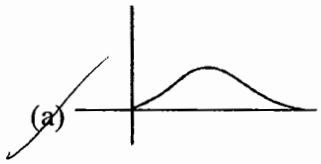
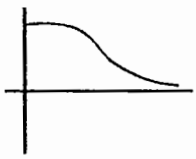
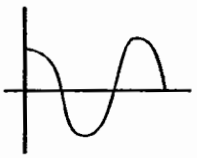
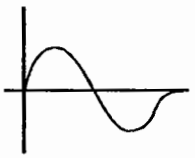
24. A function of the form $\left(x^2-2\right)e^{-ax^2}$ can represent the state of a one dimensional quantum mechanical system in

(a) its ground state
(b) its first excited state
(c) its second exited state
(d) a superposition of first and second excited state

25. A system is in the state $\psi=\frac{1}{2}\left(\psi_1+\sqrt{3}\,\psi_2\right)$ where ψ_1 and ψ_2 denote energy eigenstates corresponding to the eigenvalues E_1 and E_2 respectively. When a measurement of energy is made, the probability of obtaining the value E_1 is

(a) $\sqrt{3}/2$ (b) 3/4 (c) 1/2 (d) 1/4

26. Consider a particle in a one dimensional potential $V(x)=\begin{cases} x^2 \text{ for } x>0 \\ \infty \text{ for } x\le 0 \end{cases}$. The schematic form of the ground state wave function is as follows"

(a)  (b)  (c)  (d) 

27. $\psi_0(x), \psi_1(x)$ and $\psi_2(x)$ are the simple harmonic oscillator wave functions corresponding to energies $\frac{1}{2}\hbar\omega, \frac{3}{2}\hbar\omega$ and $\frac{5}{2}\hbar\omega$ respectively At t = 0 a particle is in he state

$$\psi(x)=\frac{i}{3}\psi_0(x)-\frac{\sqrt{5}}{3}\psi_1(x)+\frac{1}{\sqrt{3}}\psi_2(x).$$

If an energy measurement is made the most probable outcome is

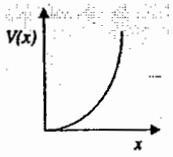
(a) $\frac{5}{2}\hbar\omega$ (b) $\frac{1}{2}\hbar\omega$ (c) $\frac{9}{2}\hbar\omega$ (d) $\frac{3}{2}\hbar\omega$

28. One-dimensional barrier is given by the potential $V(x)=\begin{cases}U \text{ for } |x|<1\\ 0 \text{ for } |x|>1\end{cases}$ where U is a positive constant.

Particles arriving from the left with an energy $E < U$ impinge on the barrier. The general form of the wave function in the region $|x|<1$ is

(a) $a\,e^{kx}+b\,e^{-kx}$ (b) $a\,e^{-ikx}+b\,e^{-kx}$ (c) $a\,e^{kx}+b\,e^{-ikx}$ (d) $a\,e^{ikx}+b\,e^{-ikx}$

29. A particle is constrained to move in a truncated harmonic potential well ($x>0$) as shown in the figure. Which one of the following statements is CORRECT ?

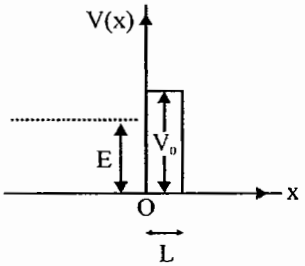


(a) The parity of the first excited state is even (b) The parity of the ground state is even

(c) The ground state energy is $\frac{1}{2}\hbar\omega$ (d) The first excited state energy is $\frac{7}{2}\hbar\omega$

Common data for Q.30, Q. 31 and Q. 32.

A beam of identical particles of mass m and energy E is incident from left on a potential barrier of width L (between $0 < x < L$) and height V_0 as shown in the figure $(E < V_0)$.



For $x > L$, there is tunneling with a transmission coefficient $T > 0$. Let A_0, A_R and A_T denote the amplitudes for the incident, reflected and the transmitted waves, respectively.

30. The ratio of the reflected to the incident amplitude $\frac{A_R}{A_0}$ is:

(a) $1-\frac{A_R}{A_0}$ (b) $\sqrt{1-T}$ in magnitude (c) A real negative number (d) $\sqrt{\left(1-\left|\frac{A_T}{A_0}\right|^2\right)\frac{E}{V_0-E}}$

31. Throughout $0<x<L$, the wave -function
(a) Can be chosen to be real (b) Is exponentially decaying.
(c) Is generally complex (d) Is zero.

32. Let the probability current associated with the incident wave be S_0. Let R be the reflection coefficient. Then
(a) The probabiltiy current vanishes in the classically forbidden region.
(b) The probability current is TS_0 for $x > L$.
(c) For, $x<0$, the probability current is $S_0(I+R)$
(d) For $x>L$, the probability current is complex.

33. An exact measurement of the position of a simple harmonic oscillator (SHO) is made with the result $x = x_0$. [The SHO has enery levels $E_n\,(n = 0,1,2,....)$ and associated normalized wavefunctions ψ_n]. Subssequently, an exact measurement of energy E is made, using the the general notation $\Pr(E = E')$ denoting the probability that a result E' is obtained for this measurement, the following statements are written. Which one of the following statements is correct?

(a) $\Pr(E = E_0) = 0$ (b) $\Pr(E = E_n) = 1$ for some value of n.

(c) $\Pr(E = E_n) \propto \psi_n(x)$ (d) $\Pr(E > E'') > 0$ for any E"

34. A particle is placed in a one dimensional box of size L along the x-axis (0<x<L). Which of the following is true?

(a) In the ground state, the probability of finding the particle in the interval $\left(\frac{L}{4}, \frac{3L}{4}\right)$ is half.

(b) In the first excited state, the probability of finding the particle in the interval $\left(\frac{L}{4}, \frac{3L}{4}\right)$ is half. This also holds for states with n = 4, 6, 8, ...

(c) For an arbitory state $|\psi\rangle$, the probability of finding the particle in the left half of the well is half.

(d) In the ground state, the particle has a definite momentum.

Common data for Q. 35, Q. 36 and Q. 37.

A particle of mass m is confined in the ground state of a one-diemensional box, extending from $x = -2L$ to $x = +2L$. The wavefunction of the particle in this state is $\psi(x) = \psi_0 \cos\frac{\pi x}{4L}$. Where ψ_0 is a constant.

35. The normalization factor ψ_0 of this wavelength is:

(a) $\sqrt{\frac{2}{L}}$ (b) $\sqrt{\frac{1}{4L}}$ (c) $\sqrt{\frac{1}{2L}}$ (d) $\sqrt{\frac{1}{L}}$

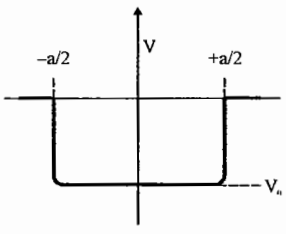
36. The energy eigenvalue corresponding to this state is:

(a) $\frac{\hbar^2\pi^2}{2mL^2}$ (b) $\frac{\hbar^2\pi^2}{4mL^2}$ (c) $\frac{\hbar^2\pi^2}{16mL^2}$ (d) $\frac{\hbar^2\pi^2}{32mL^2}$

37. The expectation value of p^2 (p is the momentum operator) in this state is:

(a) 0 (b) $\frac{\hbar^2\pi^2}{32\,L^2}$ (c) $\frac{\hbar^2\pi^2}{16\,L^2}$ (d) $\frac{\hbar^2\pi^2}{8\,L^2}$

38. There are only three bound states for a particle of mass m in a one-dimensional potential well of the form shown in the figure. The depth V_0 of the potential satisfies



(a) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{9\pi^2\hbar^2}{2ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{2\pi^2\hbar^2}{ma^2}$

(c) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{8\pi^2\hbar^2}{ma^2}$ (d) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{50\pi^2\hbar^2}{ma^2}$

39. The energy levels of a particle of mass m in a potential of the form

$$V(x) = \infty, \qquad x \le 0$$
$$= \frac{1}{2}m\omega^2 x^2, \qquad x > 0$$

are given, in terms of quantum number $n = 0, 1, 2, 3.....$, by

(a) $\left(n + \frac{1}{2}\right)\hbar\omega$ (b) $\left(2n + \frac{1}{2}\right)\hbar\omega$ (c) $\left(2n + \frac{3}{2}\right)\hbar\omega$ (d) $\left(n + \frac{3}{2}\right)\hbar\omega$

Common data for Q. 40 and Q. 41.

A particle is confined to the region $0 < x < L$ in one dimension.

40. If the particle is in the first excited state, then the probability of finding the particle is maximum at

(a) $x = \frac{L}{6}$ (b) $x = \frac{L}{2}$ (c) $x = \frac{L}{3}$ (d) $x = \frac{L}{4}$ and $\frac{3L}{4}$

41. If the particle is in the lowest energy state, then the probability of finding the particle in the region 0 to $\frac{L}{4}$ is:

(a) $\frac{1}{4} - \frac{1}{(2\pi)}$ (b) $\frac{1}{4}$ (c) $\frac{1}{4} + \frac{1}{(2\pi)}$ (d) $\frac{1}{2}$

Common data for Q. 42 and Q. 43.

The normalized wave function ψ_1 and ψ_2 correspond to the ground state and the first excited state of a particle in a potential. You are given the information that the operator $\hat{A}$ acts on the wave functions as $\hat{A}\psi_1 = \psi_2$ and $\hat{A}\psi_2 = \psi_1$

42. The expectation value of A for the state $\psi = (3\psi_1 + 4\psi_2)/5$ is:

(a) –0.32 (b) 0.0 (c) 0.75 (d) 0.96

43. Which of the following are eigenfunctions of $\hat{A}^2$?

(a) ψ_1 and ψ_2 (b) ψ_2 and not ψ_1 (c) ψ_1 and not ψ_2 (d) neither ψ_1 and ψ_2

44. The energy density of states of an electron in a one dimensional potential well of infinitely high walls is (the symbols have their usual meaning)

(a) $L\sqrt{m}/\left[\pi\hbar\sqrt{(2E)}\right]$ (b) $Lm/\left(\pi\hbar\sqrt{E}\right)$

(c) $Lm/\left[\pi\hbar\sqrt{(2E)}\right]$ (d) $L\sqrt{m}/(2\pi\hbar E)$

45. A free particle with energy E whose wave-function is a plane wave with wavelength λ enters a region of constant potential V>0 where the wavelength of the particle is 2λ. The ratio (V/E) is:

(a) 1/2 (b) 2/3 (c) 3/4 (d) 4/5

46. If the wave function of a particle trapped in space between x = 0 and x = L is given by $\psi(x) = A\sin\left(\frac{2\pi x}{L}\right)$, where A is a constant, for which value(s) of x will the probability of finding the particle be the maximum?

(a) $\frac{L}{4}$ (b) $\frac{L}{2}$ (c) $\frac{L}{6}$ and $\frac{L}{3}$ (d) $\frac{L}{4}$ and $\frac{3L}{4}$

47. Let a particle move in a potential field given by

$$V(x)=\begin{cases}\left(\frac{1}{2}\right)m\omega^2x^2 & \text{for } x>0\\ \infty & \text{for } x\le 0\end{cases}$$

The allowed energies of this particle are

(a) $(n+1/2)\hbar\omega$ (b) $(2n+3/2)\hbar\omega$ (c) $(2n+1/2)\hbar\omega$ (d) $(n+5/2)\hbar\omega$

where 'n' is a non-negative integer.

48. At $t=0$, a one dimensional harmonic oscillator of frequency ω is in a state given by

$$\psi(x,0)=\frac{1}{2}\mu_0(x)+i\sqrt{3}/2\,\mu_1(x)$$

where u_0 and u_1 are the first two normalized eigenfunctions.

(a) The probability of finding the oscillator in state u_1 is $\frac{3}{4}$.

(b) The probability of finding the oscillator in the state u_0 is $\frac{1}{2}$.

(c) The average energy of the oscillator is $2\,\hbar\omega$.

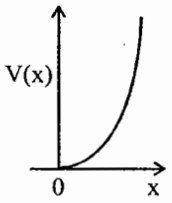
(d) The average energy of the oscillator is $(5\hbar)$.

49. A particle of mass m is confined in the potential, $V(x)=\begin{cases}\frac{1}{2}m\omega^2x^2 & \text{for } x>0,\\ \infty & \text{for } x\le 0\end{cases}$

Let the wavefunction of the particle be given by

$$\psi(x)=-\frac{1}{\sqrt{5}}\psi_0+\frac{2}{\sqrt{5}}\psi_1,$$

where ψ_0 and ψ_1 are the eigenfunctions of the ground state and the first excited state respectively. The expectation value of the energy is:



(a) $\frac{31}{10}\hbar\omega$ (b) $\frac{25}{10}\hbar\omega$ (c) $\frac{13}{10}\hbar\omega$ (d) $\frac{11}{10}\hbar\omega$

Statement for Linked Answer type Q. 50 and Q. 51.

Positronium is an atom formed by an electron and a positron. The mass of a positron is the same as that of an electron and its charge is equal in magnitude but opposite in sign to that of an electron. The positronium atom is thus similar to the hydrogen atom with the positron replacing the proton.

50. The binding energy of a positronium atom is:

(a) 13.6 eV (b) 6.8 eV (c) 27.2 eV (d) 3.4 eV

51. If a positronium atom makes a transition from the state with $n=3$ to a state with $n=2$, the energy of the photon that is emitted in this transition is closest to

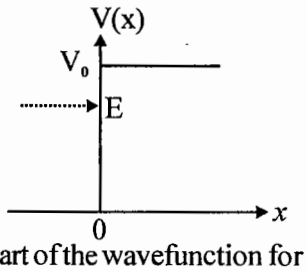
(a) 1.88 eV (b) 0.94 eV (c) 1.13 eV (d) 2.27 eV

52. An electron with energy E is incident from left on a potential barrier, given by

$$V(x)=0 \quad \text{for} \quad x<0$$

$$=V_0 \text{ for } x>0$$

as shown in the figure



For $E < V_0$, the space part of the wavefunction for $x > 0$ is of the form

(a) $e^{\alpha x}$ (b) $e^{-\alpha x}$ (c) $e^{i\alpha x}$ (d) $e^{-i\alpha x}$

where α is a real positive quantity.

Statement for Linked Answer type Q.53 and Q.54.

In a one-dimensional harmonic oscillator, φ_0, φ_1 and φ_2 are respectively the ground, first and the second excited states. These three states are normalized and are orthogonal to one another. ψ_1 and ψ_2 are two states defined by $\psi_1 = \varphi_0 - 2\varphi_1 + 3\varphi_2; \psi_2 = \varphi_0 - \varphi_1 + \alpha\varphi_2$. Where α is a constant.

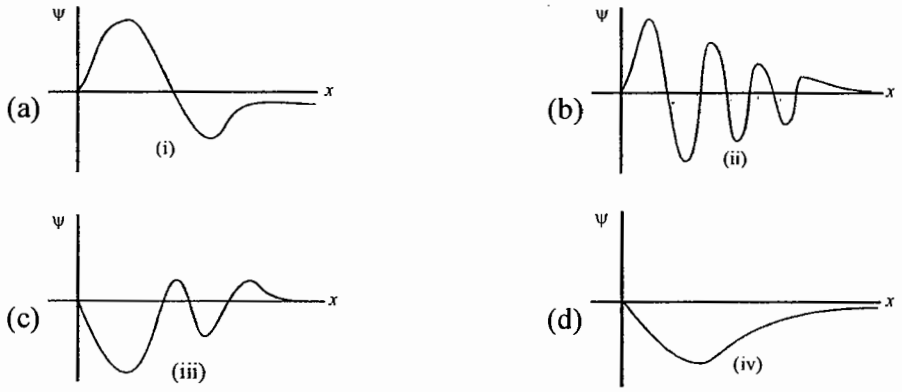
53. The value of α for which ψ_2 is orthogonal to ψ_1 is:

(a) 2 (b) 1 (c) -1 (d) -2

54. For the value of α determined in Q.13, the expectation value of energy of oscillator in the state ψ_2 is:

(a) $\hbar\omega$ (b) $3\hbar\omega/2$ (c) $3\hbar\omega$ (d) $9\hbar\omega/2$

55. A quantum particle in one dimension x moves under the influence of a potential that supports bound states. Plotted below are the wave functions ψ vs x corresponding to four eigenstates of energy. Identify the wave functions corresponding to the ground state and the highest energy state among those shown:



(a) (i) corresponds to the ground state and (iii) to the highest energy state.
(b) (ii) corresponds to the ground state and (iv) to the highest energy state.
(c) (iv) corresponds to the ground state and (ii) to the highest energy state.
(d) None of these is the ground state and (ii) corresponds to the highest energy state.

56. The ground state energy of a particle in the one-dimensional potential

$$V(x) = \begin{cases} \frac{1}{2}mw^2x^2 & \text{for } x > 0 \\ \infty & \text{for } x < 0 \end{cases}$$

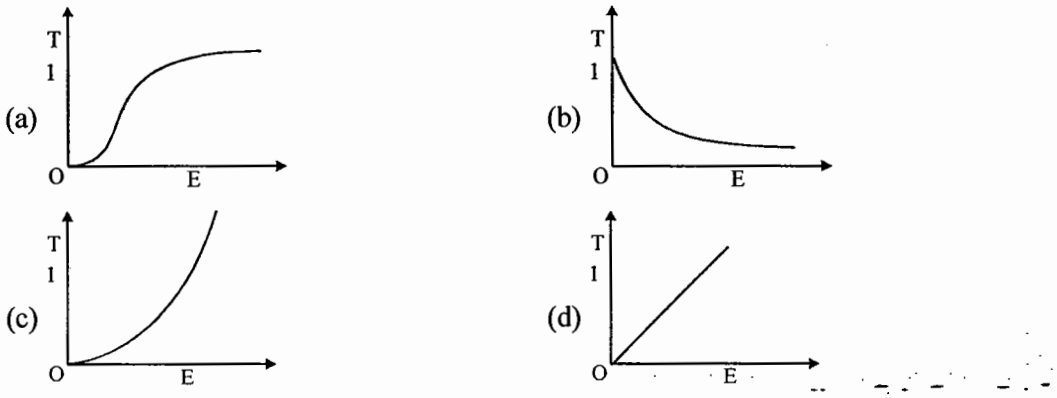
is equal to

(a) $\hbar\omega$ (b) $\frac{3}{2}\hbar\omega$ (c) $2\hbar\omega$ (d) $\frac{5}{2}\hbar\omega$

57. The particle in a box has a ground state wavefunction given by $\phi(x) = \frac{1}{\sqrt{a}}\cos\left(\frac{\pi x}{2a}\right)$. The expectation value of x is:

(a) 0 (b) $\frac{a^2}{3}$ (c) $\frac{a^2}{\pi^2}$ (d) 1

58. The variation of transmission probability with energy E, of the particles hitting a potential barrier of finite height, is represented as



59. Consider a particle trapped as a potential well. If an observer makes a measurement to detect the particle just outside the well then
(a) He will always find the particle (b) He will never find the particle
(c) He will find the particle half the timer
(d) He will find the particle only if the measurement is performed for sufficiently long time.

60. The matter wave corresponding to a free particle moving in one dimension satisfies the time-dependent Schrodinger wave equation.

$$i\hbar\frac{\delta\psi(x,t)}{\delta t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$$

If $\hbar k$ is the momentum, then the wavepacket formed out of the matter waves moves with a group velocity V_g equal to

(a) $\frac{\hbar}{2m}k^2$ (b) $\frac{\hbar^2}{2m}k^2$ (c) $\frac{\hbar^2}{2m}k$ (d) $\frac{\hbar}{2m}k$

61. A particle limited to the x-axis has the wave function $\psi = \sqrt{3}\,x$ between x = 0 and $x = 1$; $\psi = 0$ elsewhere. The expectation value $\langle x\rangle$ of the particle's position is:

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{4}$

62. The ground state energy of 5 identical spin-1/2 particles which are subject to a one-dimensional simple harmonic oscillator potential of frequency ω is

(a) $(15/2)\hbar\omega$ (b) $(13/2)\hbar\omega$ (c) $(1/2)\hbar\omega$ (d) $5\hbar\omega$

63. The wave function of free particle in one dimension is given by $\psi(x) = A\sin x + B\sin 3x$. Then $\psi(x)$ is an eigenstate of
(a) the position operator (b) the Hamiltonian
(c) the momentum operator (d) the party operator

64. A particle in one-dimension is in the potental

$$V(x)=\begin{cases}\infty & \text{if } x<0\\ -V_0 & \text{if } 0\le x\le \ell\\ 0 & \text{if } x>\ell\end{cases}$$

If there is at least one bound state, the minimum depth of the potential is:

(a) $\frac{\hbar^2\pi^2}{8m\ell^2}$ (b) $\frac{\hbar^2\pi^2}{2m\ell^2}$ (c) $\frac{2\hbar^2\pi^2}{m\ell^2}$ (d) $\frac{\hbar^2\pi^2}{m\ell^2}$

65. The displacement of a one dimensional oscillator is given by $x = A\cos(\omega t+\phi)$. The probability that the displacement lies between x and $x+dx$ is

(a) $\frac{1}{\pi\sqrt{A^2-x^2}}dx$ (b) $\frac{\pi\sqrt{A^2-x^2}}{A}dx$ (c) $\frac{A}{\pi(A^2-x^2)}dx$ (d) $\frac{1}{\pi A}dx$

66. The minimum energy of a collection of 6 non-interacting electrons of spin $-\frac{1}{2}$ placed in a one dimensional infinite square well potential of width L is

(a) $14\pi^2\hbar^2/mL^2$ (b) $91\pi^2\hbar^2/mL^2$ (c) $7\pi^2\hbar^2/mL^2$ (d) $3\pi^2\hbar^2/mL^2$

67. The energies in the ground state and first excited state of a particle of mass $m=\frac{1}{2}$ in a potential V(x) are –4 and –1, respectively, (in units in which $\hbar=1$). If the corresponding wavefunctions are related by $\psi_1(x)=\psi_0(x)\sinh x$, then the ground state eigenfunction is

(a) $\psi_0(x)=\sqrt{\text{sec}\,hx}$ (b) $\psi_0(x)=\text{sec}\,hx$ (c) $\psi_0(x)=\text{sec}\,h^2\,x$ (d) $\psi_0(x)=\text{sec}\,h^3\,x$

68. Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground and the first excited states of a one-dimensional harmonic oscillator. The uncertainty Δx in the state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ is:

(a) $\Delta x=\sqrt{\hbar/2m\omega}$ (b) $\Delta x=\sqrt{\hbar/m\omega}$ (c) $\Delta x=\sqrt{2\hbar/m\omega}$ (d) $\Delta x=\sqrt{\hbar/4m\omega}$

69. Consider the normalized state $|\psi\rangle$ of a particle in a one-dimensional harmonic oscillator: $|\psi\rangle=b_1|0\rangle+b_2|1\rangle$ where $|0\rangle$ and $|1\rangle$ denote the ground and first excited states respectively, and b_1 and b_2 are real constants. The expectation value of the displacement x in the state $|\psi\rangle$ will be a minimum when

(a) $b_2 = 0, (b_1 = 1)$ (b) $b_2=\frac{1}{\sqrt{2}}b_1$ (c) $b_2=\frac{1}{2}b_1$ (d) $b_2 = b_1$

70. A quantum mechanical particle in a harmonic oscillator potential has the initial wavefunction $\psi_0(x)+\psi_1(x)$, where ψ_0 and ψ_1 are the real wavefunction in the ground and first excited states of the harmonic oscillator Hamiltonian. For convenience we take $m = \hbar = \omega = 1$ for oscillator. What is the probability density of finding the particle at x at time $t = \pi$?

(a) $(\psi_1(x)-\psi_0(x))^2$ (b) $(\psi_1(x))^2-(\psi_0(x))^2$

(c) $(\psi_1(x)+\psi_0(x))^2$ (d) $(\psi_1(x))^2+(\psi_0(x))^2$

71. A particle is in the ground state of an infinite square well potential given by,

$$V(x)=\begin{cases}0 & \text{for } -a\le x\le a\\ \infty & \text{otherwise}\end{cases}$$

The probability to find the particle in the interval between $-\frac{a}{2}$ and $\frac{a}{2}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{2}+\frac{1}{\pi}$ (c) $\frac{1}{2}-\frac{1}{\pi}$ (d) $\frac{1}{\pi}$

72. A one dimensional harmonic oscillator is in the superposition of number state, $|n\rangle$, given by

$$|\psi\rangle=\frac{1}{2}|2\rangle+\frac{\sqrt{3}}{2}|3\rangle$$

The average energy of the oscillator in the given state is _____ $\hbar\omega$.

73. A particle in the infinite square well

$$V(x)=\begin{cases}0 & 0<x<a\\ \infty & \text{otherwise}\end{cases}$$

is prepared in a state with the wavefunction

$$\psi(x)=\begin{cases}A\sin^3\left(\frac{\pi x}{a}\right) & 0<x<a\\ 0 & \text{otherwise}\end{cases}$$

The expectation value of the energy of the particle is

(a) $\frac{5\hbar^2\pi^2}{2ma^2}$ (b) $\frac{9\hbar^2\pi^2}{2ma^2}$ (c) $\frac{9\hbar^2\pi^2}{10ma^2}$ (d) $\frac{\hbar^2\pi^2}{2ma^2}$

Assignment -4: One-dimensional Potential

[ANSWERS]

1. (b)	2. (c)	3. (d)	4. (c)	5. (a)	6. (c)	7. (d)
8. (c)	9. (b, c)	10. (c)	11. (c)	12. (b)	13. (b)	14. (b)
15. (a)	16. (d)	17. (a)	18. (c)	19. (b)	20. (c)	21. (a)
22. ( )	23. (d)	24. (c)	25. (d)	26. (d)	27. (d)	28. (a)
29. (d)	30. (d)	31. (b)	32. (b)	33. (b)	34. (c)	35. (c)
36. (d)	37. (c)	38. (a)	39. (c)	40. (d)	41. (c)	42. (d)
43. (a)	44. (a)	45. (c)	46. (d)	47. (b)	48. (a)	49. (a)
50. (b)	51. (b)	52. (b)	53. (c)	54. (b)	55. (c)	56. (b)
57. (a)	58. (b)	59. (b)	60. (d)	61. (d)	62. (b)	63. (d)
64. (a)	65. ( )	66. (a)	67. (c)	68. (a)	69. (d)	70. ()
71. (b)	72. (3.2 to 3.3)W		73. (c)			

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Assignment – 5: Angular Momentum Alzebra

1. $\sigma_i\,(i=1,2,3)$ represents the Pauli spin matrices. Which one of the following is NOT true?

   (a) $\sigma_i\sigma_j+\sigma_j\sigma_i=2\delta_{ij}$ (b) $\mathrm{Tr}(\sigma_i)=0$

   (c) The eigenvalues of σ_i are ±1 (d) $\det(\sigma_i)=1$

2. Let $|\psi_0\rangle$ denote the ground state of the hydrogen atom. Choose the correct statement from those given below

   (a) $[L_x,L_y]|\psi_0\rangle=0$ (b) $J^2|\psi_0\rangle=0$ (c) $[\vec{L},\vec{S}]|\psi_0\rangle\neq0$ (d) $[S_x,S_y]|\psi_0\rangle=0$

3. The commutator $[L_x,y]$, where L_x is the x-component of the angular momentum operator and y is the y-component of the position operator, is equal to:

   (a) 0 (b) $i\hbar x$ (c) $i\hbar y$ (d) $i\hbar z$

4. Which one of the following relations is true for Pauli matrices

   (a) $\sigma_x\sigma_y=\sigma_y\sigma_x$ (b) $\sigma_x\sigma_y=\sigma_z$ (c) $\sigma_x\sigma_y=i\sigma_z$ (d) $\sigma_y\sigma_x=i\sigma_z$

5. For a spin 1/2 particle, the expectation value of $s_xs_ys_z$. where s_x,s_y and s_z are spin operators, is

   (a) $\frac{i\hbar^3}{8}$ (b) $-\frac{i\hbar^3}{8}$ (c) $\frac{i\hbar^3}{16}$ (d) $-\frac{i\hbar^3}{16}$

6. The commutator, $[L_z,Y_{\ell m}(\theta,\phi)]$ where L_z is the z component of the orbital angular momentum and $Y_{\ell m}(\theta,\phi)$ is a spherical harmonic, is:

   (a) $\ell(\ell+1)\hbar$ (b) $-m\hbar$ (c) $m\hbar$ (d) $+\ell\hbar$

7. In a Stern-Gerlach experiment, the magnetic field is in +z direction. A particle comes out of this experiment in $|+\hat{Z}\uparrow\rangle$ state. Which of the following statements is true?

   (a) The particle has a definite value of the y-component of the spin angular momentum.
   (b) The particle has a definite value of the square of the spin angular momentum.
   (c) The particle has a definite value of x-components of spin angular momentum.
   (d) The particle has definite values of x- and y-components of spin angular momentum.

8. Let $\vec{L}=(L_x,L_y,L_z)$ denote the orbital angular momentum operators of a particle and let $L_+=L_x+iL_y$ and $L_-=L_x-iL_y$. The particle is in an eigenstate of L^2 and L_z eigenvalues $\ell(\ell+1)\hbar^2$ and $\hbar\ell$ respectively. The expectation value of $L_+\,L_-$ in this state is:

   (a) $\ell\,\hbar^2$ (b) $2\,\ell\,\hbar^2$ (c) 0 (d) $\ell\,\hbar$

9. A spin ½ particle with g>0, is subjected to a magnetic field $\vec{H}=H_0\hat{x}$. If the quantization axis is along $+\hat{z}$, then the minimum energy eigenstate is given by,

   (a) $|\downarrow\rangle$ (b) $\frac{1}{\sqrt{2}}|\uparrow\rangle-|\downarrow\rangle$ (c) $|\uparrow\rangle$ (d) $\frac{1}{\sqrt{2}}|\uparrow\rangle+|\downarrow\rangle$

10. If L_x, L_y and L_z are respectively the x, y and z components of angular momentum operator L, the commutator $\left[L_xL_y, L_z\right]$ is equal to

(a) $i\hbar\left(L_x^2+L_y^2\right)$ (b) $2i\hbar L_z$ (c) $i\hbar\left(L_x^2-L_y^2\right)$ (d) 0

11. The classical definition of orbital angular momentum of a particle is $\vec{L}=\vec{r}\times\vec{p}$. The corresponding quantum mechanical definition for the orbital angular momentum operator (taking into account the fact that $\vec{r}$ and $\vec{p}$ do not commute with each other) is:

(a) $\vec{L}=\vec{r}\times\vec{p}$ (b) $L=\frac{1}{2}\left[(\vec{r}\times\vec{p})+(\vec{p}\times\vec{r})\right]$

(c) $\vec{L}=\frac{1}{2i}\left[(\vec{r}\times\vec{p})-(\vec{p}\times\vec{r})\right]$ (d) $L=\frac{r}{r}(\vec{p}\times\vec{r})$

12. Consider L_x, L_y and L_z to be the components of the angular momentum operator $\vec{L}$. If L_+ is the raising operator, the commutator $\left[L_+, L_z\right]$

(a) $\hbar L_+$ (b) $-\hbar L_x$ (c) $-\hbar L_y$ (d) $-\hbar L_+$

13. J_x, J_y, J_z are the operators corresponding to the components of the angular momentum $\vec{J}$ and ψ_{jm} is the eigenfunction of J^2 and J_z with eigenvalues.

$$J^2\psi_{jm}=j(j+1)\hbar^2\psi_{jm}, \qquad J_z\psi_{jm}=m\hbar\psi_{jm}$$

Then the eigenvalue relation for $J_+=J_x+iJ_y$ is:

(a) $J_+\psi_{jm}=\sqrt{j(j+1)+m(m+1)}\,\hbar\psi_{jm}$ (b) $J_+\psi_{jm}=\sqrt{j(j+1)-m(m+1)}\,\hbar\psi_{jm}$

(c) $J_+\psi_{jm}=\sqrt{(j-m)}\,\hbar\psi_{jm}$ (d) $J_+\psi_{jm}=\left[\sqrt{j(j+1)}\hbar-\sqrt{m(m+1)}\hbar\right]\psi_{jm+1}$

14. P is a beam of atoms with spin quantum number $\frac{1}{2}$ and zero orbital angular momentum, with all angular momenta being along x axis. Q is a similar beam which is unpolarized. These beams can be differentiated experimentally by passing P and Q separately through Stern-Gerlach apperatus with

(a) Magnetic field along X axis (b) Magnetic field along Y axis.
(c) Magnetic field along Z axis (d) Magnetic field along YZ plane.

15. The expression $\exp i(\vec{\sigma}.\vec{n})\theta$ can be rewritten as

(a) $(\vec{\sigma}.\vec{n})\cos\theta+i\sin\theta$ (b) $\sin\theta+i(\vec{\sigma}.\vec{n})\cos\theta$

(c) $\cos\theta-i(\vec{\sigma}.\vec{n})\cos\theta$ (d) $\cos\theta+i(\vec{\sigma}.\vec{n})\sin\theta$

16. The eigenvalues of the pauli spin matrix σ_x are

(a) i, –i (b) –i, i (c) 1, –1 (d) 1, 1

17. The Clebsch-Gordon coefficient $\langle j_1m_1 : j_2m_2 \mid j_1j_2jm\rangle$ is zero unless

(a) $j=j_1+j_2$ (b) $j=j_1-j_2$ (c) $m=m_1+m_2$ (d) $m=m_1-m_2$

18. Pauli's spin operators $\sigma_x, \sigma_y, \sigma_z$ are represented by 2×2 matrices. These are
(a) All anti-hermitian and traceless (b) All orthogonal and traceless
(c) All Hermitian and traceless (d) Only σ_z is hermitian and traceless

19. The Hamiltonian of an electron in a constant magnetic field $\vec{B}$ is given by $H = \mu\vec{\sigma}.\vec{B}$ where μ is a positive constant and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ denotes the Pauli matrices. Let $\omega = \mu B/\hbar$ and I be the 2×2 unit matrix. Then the operator $e^{iHt/\hbar}$ simplifies to

(a) $I\cos\frac{\omega t}{2} + \frac{i\vec{\sigma}.\vec{B}}{B}\sin\frac{\omega t}{2}$ (b) $I\cos\omega t + \frac{i\vec{\sigma}.\vec{B}}{B}\sin\omega t$

(c) $I\sin\omega t + \frac{i\vec{\sigma}.\vec{B}}{B}\cos\omega t$ (d) $I\sin 2\omega t + \frac{i\vec{\sigma}.\vec{B}}{B}\cos 2\omega t$

20. The orbital angular momentum eigenstate $\psi_{\ell,m}(r,\theta\,\phi)$ with $l=0, m=0$ (in standard notation) is an eigenstate of the following operators
(a) L_x, L_y, L_z and L^2 (b) L_x and L^2 only (c) L_y and L^2 only (d) L_z and L^2 only

21. The Dirac Hamiltonian for the relativistic Hydrogen atom has a coupling $\vec{L}.\vec{S}$ between the orbital angular momentum and the spin of the electron. Let s_z, l_z and j_z denote the z-components of the spin, orbit and total angular momenta respectively. For the electron in an eigenstate of energy
(a) j_z and s_z (but not l_z) can be measured precisely
(b) j_z (but not s_z and l_z) can be measured precisely
(c) j_z and l_z (but not s_z) can be measured precisely
(d) l_z (but not l_z and s_z) can be measured precisely

22. which of the following describes the same physical state of a spin-1/2 particle as $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}$?

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 0\end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\ -1\end{pmatrix}$

23. If $J_+|j,m\rangle = 0$ where 'J_+' denotes the angular momentum raising operator, $J_+ = J_x + iJ_y$ the value of m is
(a) -j+1 (b) $-j$ (c) $j-1$ (d) j

24. The wavefunction of a particle is of the form $\psi(r,\theta,\phi) = u(r,\theta)\cos(\phi)$. If the z-component of the angular momentum L_z is measured, then the possible outcome is $+\hbar$ with probability
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

25. The spin function of a free particle, in the basis in which S_z is diagonal, can be written as $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ with eigenvalues $+\frac{\hbar}{2}$ and $\frac{\hbar}{2}$, respectively. In the given basis, the normalized eigenfunction of S_y with eigenvalue $-\frac{\hbar}{2}$

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\i\end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\0\end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\1\end{pmatrix}$

26. Two spin -1/2 fermions having spins $\vec{S}_1$ and $\vec{S}_2$ interact via a potential $V(r)=\vec{S}_1.\vec{S}_2V_0(r)$. The contributions of this potential in the singlet and triplet states, respectively, are

(a) $-\frac{3}{2}V_0(r)$ and $\frac{1}{2}V_0(r)$ (b) $\frac{1}{2}V_0(r)$ and $-\frac{3}{2}V_0(r)$

(c) $\frac{1}{4}V_0(r)$ and $-\frac{3}{4}V_0(r)$ (d) $-\frac{3}{4}V_0(r)$ and $\frac{1}{4}V_0(r)$

Common data for Q.27, Q. 28 and Q.29.

A system consists of three spin-half particles, the z components of whose spins $S_z(1), S_z(2)$ and $S_z(3)$ can take value $+1/2$ and $-1/2$. The total spin of the system is $S_z=S_z(1)+S_z(2)+S_z(3)$

27. The total number of possible micro-states of this system is:
(a) 3 (b) 6 (c) 7 (d) 8

28. The total number of micro-states with $S_z=1/2$ is:
(a) 3 (b) 5 (c) 6 (d) 7

29. Consider an ensemble of systems where each microstate has equal probability. The ensemble average of S_z is:
(a) $-1/2$ (b) 0 (c) 1/2 (d) 3/2

30. A spin half particle is in the state $S_z=\hbar/2$. The expectation values of S_x, S_x^2, S_y, S_y^2 are given by
(a) $0,0,\hbar^2/4,\hbar^2/4$ (b) $0,\hbar^2/4,\hbar^2/4,0$ (c) $0,\hbar^2/4,0,\hbar^2/4$ (d) $\hbar^2/4,\hbar^2/4,0,0$

31. An electron is in a state spin wave function $\phi_s=\begin{pmatrix}\sqrt{3}/2\\1/2\end{pmatrix}$ in the S_z representation. What is the probability of finding the z-componenet of its spin along the $-\hat{z}$ direction?
(a) 0.75 (b) 0.50 (c) 0.35 (d) 0.25

32. If a spin I particle is in the state $|m=0\rangle$ with respect to a quantization axis $\hat{n}$, which of the following is correct?
(a) $\langle\vec{S}\rangle=0$ (b) $\langle\vec{S}\rangle=\hat{n}$ (c) $\langle\vec{S}\rangle=\sqrt{2}\hat{n}$ (d) $\langle\vec{S}\rangle=-\hat{n}$

33. The total spin-angular momentum of a system of three free electrons is:
(a) $S_{tot}=\frac{1}{2}$ and $S_{tot}=\frac{3}{2}$ (b) $S_{tot}=\frac{1}{2}$ only
(c) $S_{tot}=\frac{3}{2}$ only (d) $S_{tot}=0$ $S_{tot}=1$

Statement for Linked Answer type Q. 34 and Q. 35.

The state $|m>, m=-1,0,1$ are the eigenstates of S_z, the z component of the spin angular momentum of a particle with $S=1.\left[S_z|m\rangle = m\hbar|m\rangle\right]$

34. The expectation value of S_z in the state $|\psi\rangle = \frac{1}{2}|1\rangle + \frac{1}{2}|0\rangle + \frac{1}{\sqrt{2}}|-1\rangle$ is:

(a) $\hbar/4$ (b) $-\hbar/2$ (c) $\hbar/2$ (d) $-\hbar/4$

35. Consider the states $|\psi_1\rangle = \frac{1}{\sqrt{6}}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{3}}|-1\rangle$ and $|\psi_2\rangle = -\frac{1}{\sqrt{6}}|1\rangle + a|0\rangle - \frac{1}{\sqrt{3}}|-1\rangle$. The value of a for which these states are orthogonal is:

(a) $1/\sqrt{3}$ (b) $1/2$ (c) $1/\sqrt{2}$ (d) $-1/\sqrt{2}$

Statement for Linked Answer type Q. 36 and Q. 37.

An electron is placed in an uniform magnetic field $\vec{H}$ that points in the $\hat{x}$ direction. The Hamiltonian of the system is $H = -K\vec{S}.\vec{H}$ where $k>0$ is a constant and $\vec{S}$ is the electron spin operator.

36. The energy of the ground state of the Hamiltonian is:

(a) $-KS\hbar$ (b) $\frac{1}{2}KH\hbar$ (c) $-\frac{1}{2}KH\hbar$ (d) 0

37. The expectation value of S_z in the ground state of this Hamiltonian is:

(a) $-\hbar/2$ (b) 0 (c) $\hbar/2$ (d) $\hbar$

38. The eigenstates of $\hat{S}_z$ of a quantum particle with spin $\frac{1}{2}$ are $|\uparrow\rangle = \left|s=\frac{1}{2}, s_z=\frac{1}{2}\right\rangle$ and $|\downarrow\rangle = \left|s=\frac{1}{2}, s_z=-\frac{1}{2}\right\rangle$.

Which of the following statements about eigenstates of the operator $\hat{S}_x$ is true?

(a) $|\uparrow\rangle$ is an eigenstate of $\hat{S}_x$. (b) $|\downarrow\rangle$ is an eigenstate of $\hat{S}_x$

(c) $|\uparrow\rangle + |\downarrow\rangle$ is an eigenstate of $\hat{S}_x$

(d) No linear combination of $|\uparrow\rangle$ and $|\downarrow\rangle$ can be an eigenstate of $\hat{S}_x$

39. Consider a system of two spin-half particles, in a state with total spin quantum number $S=0$. The eigenvalue of the spin Hamiltonian $H = A\,\vec{S}_1\vec{S}_2$ (A is a positive constant) in this state is:

(a) $A\hbar^2/4$ (b) $-A\hbar^2/4$ (c) $3A\hbar^2/4$ (d) $-3A\hbar^2/4$

40. The eigenvalues of the Pauli spin matrix $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ are

(a) $+1,+1$ (b) $+1,-1$ (c) $-i,+i$ (d) $+i,+i$

41. In a system consisting of two spin $\frac{1}{2}$ particles labeled 1 and 2, let $\vec{S}^{(1)} = \frac{\hbar}{2}\vec{\sigma}^{(1)}$ and $\vec{S}^{(2)} = \frac{\hbar}{2}\vec{\sigma}^{(2)}$ denote the corresponding spin operators. Here $\vec{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$ and $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices.

A. In the standard basis the matrices for the operators $S_x^{(1)}\,S_y^{(2)}$ and $S_y^{(1)}S_x^{(2)}$ respectively,

(a) $\frac{\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$ (b) $\frac{\hbar^2}{4}\begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}$

(c) $\frac{\hbar^2}{4}\begin{pmatrix}0 & 0 & 0 & -i\\ 0 & 0 & i & 0\\ 0 & -i & 0 & 0\\ i & 0 & 0 & 0\end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix}0 & 0 & 0 & -i\\ 0 & 0 & -i & 0\\ 0 & i & 0 & 0\\ i & 0 & 0 & 0\end{pmatrix}$ (d) $\frac{\hbar^2}{4}\begin{pmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & -i\\ 0 & 0 & i & 0\end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix}0 & -i & 0 & 0\\ i & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$

B. These two operators satisfy the relation

(a) $\left\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\right\} = S_z^{(1)}S_z^{(2)}$ (b) $\left\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\right\} = 0$

(c) $\left[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\right] = i\,S_z^{(1)}S_z^{(2)}$ (d) $\left[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\right] = 0$

42. Consider a spin- 1/2 particle in the presence of a homogeneous magnetic field of magnitude B long along z-axis which is prepared initially in a state $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$ at time $t = 0$. At what time t will the particle be in the state $-|\psi\rangle$ (μ_B is Bhor magneton) ?

(a) $t = \frac{\pi\hbar}{\mu_B B}$ (b) $t = \frac{2\pi\hbar}{\mu_B B}$ (c) $t = \frac{\pi\hbar}{2\mu_B B}$ (d) Never

43. The quantum state $\sin x|\uparrow\rangle + \exp(i\phi)\cos x|\downarrow\rangle$, where $\langle\uparrow|\downarrow\rangle = 0$ and x, ϕ are real is orthogonal to :

(a) $\sin x|\uparrow\rangle$ (b) $\cos x|\uparrow\rangle + \exp(i\phi)\sin x|\downarrow\rangle$

(c) $-\cos x|\uparrow\rangle - \exp(i\phi)\sin x|\downarrow\rangle$ (d) $-\exp(-i\phi\cos x)|\uparrow\rangle + \sin x|\downarrow\rangle$

44. The component along an arbitrary direction $\hat{n}$, with direction cosines (n_x, n_y, n_z), of the spin of a spin $-\frac{1}{2}$ particle is measured. The result is:

(a) 0 (b) $\pm\frac{\hbar}{2}n_z$ (c) $\pm\frac{\hbar}{2}(n_x, n_y, n_z)$ (d) $\pm\frac{\hbar}{2}$

45. Which one of the following commutation relations is NOT CORRECT? Here, symbols have their usual meanings.

(a) $[L^2, L_z] = 0$ (b) $[L_x, L_y] = i\hbar L_z$ (c) $[L_z, L_+] = \hbar L_+$ (d) $[L_z, L_-] = \hbar L_-$

46. A spin-half particle is in a linear superposition $0.8|\uparrow\rangle + 0.6|\downarrow\rangle$ of its spin-up and spin-down states. If $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigen states of σ_z then what is the expectation value, up to one decimal place, of the operator $10\sigma_z + 5\sigma_x$? Here, symbols have their usual meanings _____

47. In a basis in which the z-component S_z of the spin is diagonal, an electron is in a spin state

$$\psi = \begin{pmatrix} (1+i)/\sqrt{6} \\ \sqrt{2/3} \end{pmatrix}$$

The probabilities that a measurement of S_z will yield the values $\hbar/2$ and $-\hbar/2$ are, respectively.

(a) 1/2 and 1/2 (b) 2/3 and 1/3 (c) 1/4 and 3/4 (d) 1/3 and 2/3

48. If J_x, J_y, J_z are angular momentum operators, the possible eigenvalues of the operator $(J_x + J_y)/\hbar$ are

(a) real and discrete with rational spacing
(b) real and discrete with irrational spacing
(c) real and continuous
(d) not all real

49. Consider the state $\begin{pmatrix} 1/2 \\ 1/2 \\ 1/\sqrt{2} \end{pmatrix}$ corresponding to the angular momentum $l = 1$ in the L_z basis of states with $m = +1, 0, -1$. If L_z^2 is measured in this state yielding a result 1, what is the state after the measurement?

(a) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1/\sqrt{3} \\ 0 \\ \sqrt{2/3} \end{pmatrix}$ (c) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

50. What are the eigenvalues of the operator $H = \vec{\sigma} \cdot \vec{a}$, where $\vec{\sigma}$ are the three Pauli matrices and $\vec{a}$ is a vector?

(a) $a_x + a_y$ and a_z (b) $a_x + a_z \pm i a_y$ (c) $\pm(a_x + a_y + a_z)$ (d) $\pm|\vec{a}|$

51. The expectation value of the x-component of the orbital angular momentum L_x in the state

$\psi = \frac{1}{5}\left[3\psi_{2,1,-1} + \sqrt{5}\psi_{2,1,0} - \sqrt{11}\psi_{2,1,+1}\right]$ (where ψ_{nlm} are the eigenfunctions in usual notation), is

(a) $-\frac{\hbar\sqrt{10}}{25}(\sqrt{11}-3)$ (b) 0

(c) $\frac{\hbar\sqrt{10}}{25}(\sqrt{11}+3)$ (d) $\hbar\sqrt{2}$

52. A particle is prepared in a simultaneous eigenstate of L^2 and L_z. If $\ell(\ell+1)\hbar^2$ and $m\hbar$ are respectively the eigenvalues of L^2 and L_z, then the expectation value $\langle L_x^2 \rangle$ of the particle in this state satisfies

(a) $\langle L_x^2 \rangle = 0$ (b) $0 \le \langle L_x^2 \rangle \le \ell^2\hbar^2$

(c) $0 \le \langle L_x^2 \rangle \le \frac{\ell(\ell+1)\hbar^2}{3}$ (d) $\frac{\ell\hbar^2}{2} \le \langle L_x^2 \rangle \le \frac{\ell(\ell+1)\hbar^2}{2}$

53. ψ_1 and ψ_2 are two orthogonal states of a spin ½ system. it is given that

$$\psi_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix}$$

where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represents the spin-up and spin-down states, respectively. When the system is in the state ψ_2, its probability to be in the spin-up state is _____

[ANSWERS]

1. (d)	**2. (a)**	**3. (d)**	**4. (c)**	**5. (a)**	**6. (c)**	**7. (d)**
8. (b)	**9. (b)**	**10. (c)**	**11. (a)**	**12. (a)**	**13. (d)**	**14. (a)**
15. (d)	**16. (c)**	**17. (c)**	**18. (c)**	**19. (b)**	**20. (d)**	**21. (b)**
22. (d)	**23. (d)**	**24. (c)**	**25. (d)**	**26. (d)**	**27. (d)**	**28. (a)**
29. (b)	**30. (c)**	**31. (d)**	**32. (a)**	**33. (a)**	**34. (d)**	**35. (c)**
36. (c)	**37. (b)**	**38. (c)**	**39. (d)**	**40. (b)**	**41_A(b)**	**41_B(b)**
42. (a)	**43. (d)**	**44. (d)**	**45. (d)**	**46. (7.6)**	**47. (d)**	**48. ( )**
49. ( )	**50. ( )**	**51. (a)**	**52. (d)**	**53.** (0.66 to 0.68)		

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Assignment – 6: 3-D Potentials

1. The radial wave function of the electrons in the state of n = 1 and l = 0 in a hydrogen atom is $R_{10} = \frac{2}{a_0^{3/2}} \exp\left(-\frac{r}{a_0}\right)$, a_0 is the Bohr radius. The most probable value of r for an electron is:

(a) a_0 (b) $2a_0$ (c) $4a_0$ (d) $8a_0$

2. An atomic state of hydrogen is represented by the following wave function:

$$\psi(r,\theta,\phi) = \frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}\cos\theta$$

where a_0 is a constant. The quantum number of the state are

(a) $\ell = 0, m = 0, n = 1$ (b) $\ell = 1, m = 1, n = 2$ (c) $\ell = 1, m = 0, n = 2$ (d) $\ell = 2, m = 0, n = 3$

3. The expectation value of the z coordinate (z), in the ground state of the hydrogen atom (wavefunction: $\psi_{100}(r) = Ae^{-r/a_0}$, where A is the normalization constant and a_0 is the Bohr radius), is

(a) a_0 (b) $\frac{a_0}{2}$ (c) $\frac{a_0}{4}$ (d) 0

4. The degeneracy of the n = 2 level for a three dimensional isotropic oscillator is

(a) 4 (b) 6 (c) 8 (d)

5. Bound state eigenfunctions of an attractive finite range smooth potential behave for larger r as:

(a) $\exp(-r/r_0)$ where r_0 is a positive constant. (b) (I/r^n) where n>0

(c) constant (d) $\exp(ikr)/r$

6. Given that binding energy of hydrogen atom in the ground state is 13.6 eV, the binding energy for n = 2 state of the positronium atom is

(a) 13.6 eV (b) 6.8 eV (c) 3.4 eV (d) 1.7 eV

7. Degeneracy of the first excited state of an isolated hydrogen atom is:

(a) 2 (b) 4 (c) 6 (d) 8

8. The ground state wave function ψ of hydrogen atom in spherical polar co-ordinates has no angular dependence but only radial dependence. Then ψ is an eigen function of

(a) L_x, L_y and L_z simultaneously (b) L_z but not of L_x and L_y

(c) none of L_x, L_y and L_z (d) p_z

9. The Hamiltonian of the electron in the hydrogen atom is

$$H = p^2/(2m) - e^2/(4\pi\varepsilon_0 r)$$

where the symbols have their usual meanings. Let $|\psi\rangle$ denote the ground state of H, with $\langle\psi|\psi\rangle = 1$

(a) $|\psi\rangle$ is an eigen state of $p^2/(2m)$ (b) $\langle\psi|r|\psi\rangle$ is zero

(c) $\langle\psi|(1/r)|\psi\rangle$ is infinite (d) $\langle\psi|H|\psi\rangle < 0$

10. The ground state energy of the Hydrogen atom is –13.6 eV. The energy of the second excited state is:
(a) –27.2 eV (b) –6.8 eV (c) –1.5 eV (d) –4.5 eV

11. The degeneracy of energy for a hydrogen-like atom with principal quantum number n and orbital quantum number l is:
(a) n (b) n^2 (c) $2n^2$ (d) $l(l+1)$

12. In a central force field, the angular part of the solution of the Schrodinger equation is in general
(a) Independent of θ and ϕ (b) A constant
(c) A spherical harmonic (d) A Bessel function

13. Consider 1s and 2s states for hydrogen atom, then $\int \psi_{100}(\vec{r})\psi_{200}(\vec{r})\, d^3r$, is
(a) –ve (b) +ve (c) zero (d) 1.0

14. The wave function $\psi(\vec{r}) = e^{i\vec{p}.\vec{r}}$ is an eigen function for a Hamiltonian with
(a) A constant potential $V \geq 0$
(b) A zero potential only
(c) A potential V(r) (not constant) which is spherically symmetric.
(d) Any potential which is periodic with periodicity $\vec{R}$.

15. The azimuthal wave function for the hydrogen atom is: $\phi(\varphi) = Ae^{im\phi}$. Where m is the magnetic quatum number. If $\phi(\varphi)$ is normalized, the value of the normalization constant A is:
(a) $\frac{1}{2\sqrt{\pi}}$ (b) $\frac{1}{2\sqrt{m\pi}}$ (c) $\frac{1}{\sqrt{2\pi}}$ (d) $\frac{1}{\sqrt{2m\pi}}$

16. The energy levels of the non-relativistic electron in a hydrogen atom (i.e. in a Coulomb potential $V(r) \propto -1/r$ are given by $E_{n\ell m} \propto -1/n^2$, where n is the principal quantum number, and the corresponding wave functions are given by $\psi_{n\ell m}$, where ℓ is the orbital angular momentum quantum number and m is the magnetic quantum number. The spin of the electron is not considered. Which of the following is a correct statement?
(a) There are exactly $(2\ell+1)$ different wave functions $\psi_{n\ell m}$, for each $E_{n\ell m}$.
(b) There are $\ell(\ell+1)$ different wave functions $\psi_{n\ell m}$, for each $E_{n\ell m}$.
(c) $E_{n\ell m}$ does not depend on ℓ and m for the Coulomb potential.
(d) There is a unique wave function $\psi_{n\ell m}$ and $E_{n\ell m}$.

17. Consider the high excited states of a Hydrogen atom corresponding to large values of the principal quantum number $(n \gg 1.)$. The wavelength λ of a photon emitted due to an electron undergoing transition between two such states with consecutive values of n (i.e., $\psi_{n+1} \rightarrow \psi_n$) is related to the wavelenth λ_α of the K_α line of Hydrogen by
(a) $\lambda = n^3\lambda_\alpha/8$ (b) $\lambda = 3n^3\lambda_\alpha/8$ (c) $\lambda = n^2\lambda_\alpha$ (d) $\lambda = 4\lambda_\alpha/n^2$

18. The normalized wavefunctions of a Hydrogen atom are denoted by $\psi_{n,l,m}(\vec{x})$, where n, l and m are, respectively, the principal, azimuthal and magnetic quantum numbers respectively. Now consider an electron in the mixed state

$$\psi(\vec{x}) = \frac{1}{3}\psi_{1,0,0,}(\vec{x}) + \frac{2}{3}\psi_{2,1,0}(\vec{x}) + \frac{2}{3}\psi_{3,2,-2}(\vec{x})$$

The expectation value $\langle E \rangle$ of the energy of this electron, in electron-Volts (eV) will be approximately

(a) -1.5 (b) -3.7 (c) -13.6 (d) -80.1 (e) $+13.6$

19. The energy of the first excited quantum state of a particle in the two-dimensional potential $V(x,y) = \frac{1}{2}m\omega^2(x^2 + 4y^2)$ is:

(a) $2\hbar\omega$ (b) $3\hbar\omega$ (c) $\frac{3}{2}\hbar\omega$ (d) $\frac{5}{2}\hbar\omega$

20. If $\psi_{n,\ell,m}(\vec{r})$ denote normalised eigenfunctions of the Hamiltonian H of the hydrogen atom, then the wave function $\frac{1}{2}\left[\psi_{2,1,1}(\vec{r}) + \psi_{2,1,-1}(\vec{r})\right]$ is an eigenfunction of

(a) L^2 and L_z (b) H and L_z (c) H and L^2 (d) L_z alone

21. An electron in the hydrogen atom is in a state whose energy is -3.4 eV. The degeneracy of the state is

(a) 4 (b) 8 (c) 2 (d) 1

22. The quantum states of a particle in a Coulomb potential is labelled by the principal, orbital and azimuthal quantum numbers n, l and m respectively. From the rotational symmetry of the Hamiltoian alone we may conclude that the energy eigenvalues are

(a) independent of n only (b) independent of l only
(c) independent of l and m (d) independent of m only

23. The degeneracy of the state of energy $E_n = \hbar\omega(n+1)$, where n is an integer, in a two-dimensional isotopic harmocnic oscillator with potential $v(x,y) = \frac{1}{2}m\omega^2(x^2 + y^2)$ is

(a) $\frac{n(n-1)}{2}$ (b) $n+1$ (c) $n-1$ (d) n

24. A measurement of the z-omponent of angular momentum L_z is made for an electrons moving in the Coulomb field of a proton with wave function $\frac{1}{4}\left[\psi_{1,0,0}(\vec{r}) + 3\psi_{2,1,1}(\vec{r}) - \sqrt{6}\,\psi_{2,1,-1}(\vec{r})\right]$ where $\psi_{n,\ell,m}(\vec{r})$ are Hydrogen atom states. The expectation value of L_z is:

(a) $\frac{3}{16}\hbar$ (b) $\frac{3-\sqrt{6}}{4}\hbar$ (c) $\frac{3+\sqrt{6}}{4}\hbar$ (d) $\frac{3-\sqrt{6}}{16}\hbar$

25. If E_H is the binding energy of the electron in the Bohr model of the Hydrogen atom, and both the charge and mass of the electron are doubled, the binding energy becomes

(a) $32\,E_H$ (b) $2\,E_H$ (c) $4\,E_H$ (d) $16\,E_H$

26. A particle of mass m is confined in a two dimensional square well potential of dimension a. This potential $V(x,y)$ is given by

$$V(x,y) = 0 \text{ for } -a < x < a \text{ and } -a < y < a$$
$$= \infty \text{ else where}$$

The energy of the first excited state for this particle is given by,

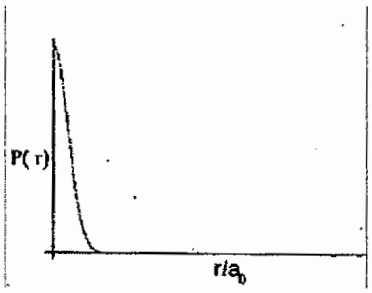
(a) $\frac{\pi^2\hbar^2}{ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2}$ (c) $\frac{5\pi^2\hbar^2}{2ma^2}$ (d) $\frac{4\pi^2\hbar^2}{ma^2}$

27. The ground state wavefunction for the hydrogen atom is given by $\Psi_{100} = \frac{1}{\sqrt{4\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$, where a_0 is the Bohr radius.

The plot of the radial probability density, P(r) for the hydrogen atom in the ground state is

(a)

(b)

(c) 

(d)

Linked Answer for Q.28 and Q.29:

The one-electron states for non-interacting electrons confined in a cubic box of side a are $\varepsilon_0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 < \varepsilon_4$ etc.

28. The energy of the lowest state is:

(a) zero (b) $\frac{\hbar^2\pi^2}{2ma^2}$ (c) $\frac{\hbar^2\pi^2}{ma^2}$ (d) $\frac{3\hbar^2\pi^2}{2ma^2}$

29. The degeneracy (including spin) of the level ε_3 is:

(a) 2 (b) 4 (c) 6 (d) 8

Common data for Q.30 and Q.31.

A particle is located in a three diamensional cubic well of width L with impenetrable walls

30. The sum of he energies of the third and the fourth levels is:

(a) $10\pi^2\hbar^2/mL^2$ (b) $10\pi^2\hbar^2/3\,mL^2$ (c) $11\pi^2\hbar^2/2mL^2$ (d) $15\pi^2\hbar^2/2mL^2$

31. The degeneracy of the fourth level is given by

(a) 1 (b) 2 (c) 3 (d) 4

32. A quantum particle of mass m is confined to a square region in xoy-plane whose vertices are given by (0, 0), (L, 0), (L, L) and (0, L). Which of the following represents an admissible wave function of the particle (for l, m, n positive integers)?

(a) $\frac{2}{L}\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi y}{L}\right)$ (b) $\frac{2}{L}\sin\left(\frac{\ell\pi x}{L}\right)\cos\left(\frac{n\pi y}{L}\right)$

(c) $\frac{2}{L}\sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right)$ (d) $\frac{2}{L}\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{\ell\pi y}{L}\right)$

33. The ground state wave function of the hydrogen atom S given by $\psi(r)=\left(1\sqrt{\pi}\,a^{3/2}\right)e^{-r/a}$ where a is a constant. If $P(r)dr$ is the probability of finding the electron between r and $r+dr$, then

(a) $P(r)=0$ at $r=0$ (b) $P(r)$ is maximum at $r=0$

(c) $P(r)$ is maximum at $r=a$ (d) $\int_0^\infty P(r)dr=1$

34. A system is known to be in a state described by the wave function, $\psi(\theta,\phi)=\frac{1}{\sqrt{30}}\left(5Y_4^0+Y_6^0-2Y_6^3\right)$ where $Y_\ell^m(\theta,\phi)$ are the spherical harmonics. The probability of finding the system in a state with $m=0$ is:

(a) zero (b) 2/15 (c) 1/4 (d) 13/15

35. A quantum particle of mass m moves in two dimensions in an anistropic harmonic oscillator potential $V(x,y)=\frac{1}{2}m\omega^2x^2+2m\omega^2y^2$. The energy eigenvalues are (n is a positive integer or zero)

(a) $\hbar\omega(2n+1)$ (b) $\hbar\omega(n+1)$ (c) $2\hbar\omega(n+1)$ (d) $\hbar\omega\left(n+\frac{3}{2}\right)$

Statement for Linked Answer type Q. 36 and Q. 37.
Consider a three dimensional harmonic oscillator with Hamiltonian

$$H=\frac{p_x^2}{2m}+\frac{p_y^2}{2m}+\frac{p_z^2}{2m}+\frac{1}{2}m\omega^2\left(x^2+y^2+z^2\right)$$

36. The number of distinct eigenvalues with energy eigenvalue $\frac{5}{2}\hbar\omega$ is:

(a) 5 (b) 3 (c) 2 (d) 1

37. The eigenvalue of L^2 (where $\vec{L}$ is the angular momentum operator) in the ground state is:

(a) $\hbar^2$ (b) $2\hbar^2$ (c) 0 (d) $6\hbar^2$

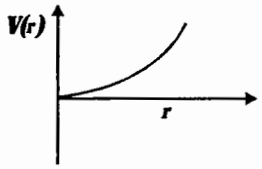
38. The normalized ground state wavefunction of a hydrogen atom is given by $\psi(r)=\frac{1}{\sqrt{4\pi}}\frac{2}{a^{3/2}}e^{-r/a}$, where a is the Bohr radius and r is the distance of the electron from the nucleus, located at the origin. The expectation value $\left\langle\frac{1}{r^2}\right\rangle$ is:

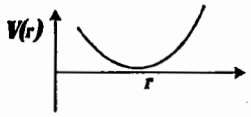
(a) $\frac{8\pi}{a^2}$ (b) $\frac{4\pi}{a^2}$ (c) $\frac{4}{a^2}$ (d) $\frac{2}{a^2}$

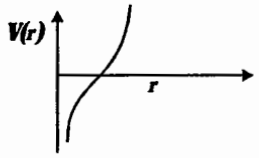
39. The number of nodes of the radial wave function for an atomic electron, $R(r) = A\left[27 - 18r + 2r^2\right]e^{-r/3}$ is:

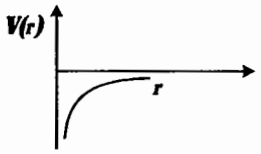
(a) 0 (b) 1 (c) 2 (d) 3

40. Which of the following figure represents the potential, $V(r) = -\frac{A}{r} + Br^2$

(a) 

(b) 

(c) 

(d) 

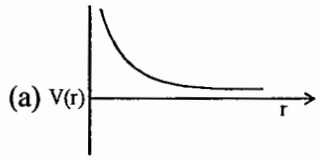
41. The electronic configuration of the ground state of a nitrogen is $1s^2\,2s^2\,2p^3$. The degeneracy of this configuration is:

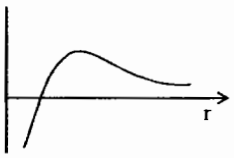
(a) 3 (b) 20 (c) 6 (d) 0

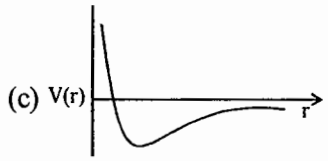
42. The effective radial potential for the motion of the electron in the hydrogen atom is of the form

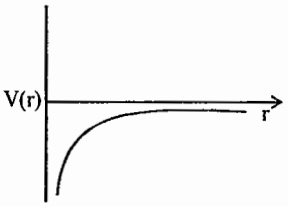
$$V(r) = -\frac{K}{r} + \frac{\ell(\ell+1)\hbar^2}{2mr^2}$$

Where K is strength of interaction. ℓ is angular momentum quantum number and m is mass of the particle. The variation of V(r) with r is given by

(a) 

(b) 

(c) 

(d) 

43. Consider a quantum particle of mass m in a three-dimensional isotropic simple harmonic potential $V(x,y,z) = \frac{1}{2}m\omega^2\left(x^2 + y^2 + z^2\right)$. It is known that the particle is in an energy eigenstate with eigenvalue $E = 7\hbar\omega/2$. Which of the following cannot be the wave function of the particle? (In the following $\alpha = \sqrt{m\omega/\hbar}$ and $H_n(\xi)$ is the n^{th} hermite polynomial).

(a) $H_2(\alpha x)\exp\left(-\alpha\left(y^2 + z^2\right)\right)$ (b) $H_2(\alpha x)\exp\left(-\alpha\left(x^2 + y^2 + z^2\right)\right)$

(c) $H_1(\alpha y)\,e\,H_1(\alpha z)\exp\left(-\alpha\left(x^2 + y^2 + z^2\right)\right)$ (d) $H_1(\alpha x)H_1(\alpha z)\exp\left(-\alpha\left(x^2 + y^2 + z^2\right)\right)$

44. If an electron is in the ground state of the hydrogen atom, the probability that its distance from the proton is more than one Bohr radius is approximately

(a) 0.68 (b) 0.48 (c) 0.28 (d) 0.91

45. Consider a particle of mass m moving inside a two- dimensional square box whose sides are described by the equations $x = 0, x = L, y = 0, y = L$. What is the lowest eigenvalue of an eigenvalue which changes sign under the exchange of x and y?

(a) $\hbar^2/(mL^2)$ (b) $3\hbar^2/(2mL^2)$ (c) $5\hbar^2/(2mL^2)$ (d) $7\hbar^2/(2mL^2)$

46. A particle of mass m is in a cubic box of size a. The potential inside the box. $(0 \le x < a, 0 \le y < a, 0 \le z < a)$ is zero and infinte outside. If the particle is in an eigenstate of energy $E = \frac{14\pi^2h^2}{2ma^2}$, its wave function is:

(a) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{3\pi x}{a}\sin\frac{5\pi y}{a}\sin\frac{6\pi z}{a}$ (b) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{7\pi x}{a}\sin\frac{4\pi y}{a}\sin\frac{3\pi z}{a}$

(c) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{4\pi x}{a}\sin\frac{8\pi y}{a}\sin\frac{2\pi z}{a}$ (d) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{\pi x}{a}\sin\frac{2\pi y}{a}\sin\frac{3\pi z}{a}$

47. Let $\Psi_{n\ell m}$ denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential V(r). The wavefunction $\Psi = \frac{1}{4}\left[\Psi_{210} + \sqrt{5}\Psi_{21-1} + \sqrt{10}\Psi_{213}\right]$ is an eigenfunction only of

(a) H, L^2 and L_z (b) H and L_z (c) H and L^2 (d) L^2 and L_z

48. The wave function of a state of the hydrogen atom is given by

$$\psi = \psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\,\psi_{21-1}$$

where $\psi_{n\ell m}$ denotes the normalized eigen function of the state with quantum numbers n, l and m in the usual notation. The expectation value of L_z in the state ψ is:

(a) $\frac{15\hbar}{16}$ (b) $\frac{11\hbar}{16}$ (c) $\frac{3\hbar}{8}$ (d) $\frac{\hbar}{8}$

49. A muon (μ^-) from cosmic rays is trapped by a proton to form a hydrogen-like atom. Given that a muon is approximately 200 times heavier than an electron, the longest wavelength of the spectral line (in the analogue of the Lyman series) of such an atom will be

(a) 5.62 Å (b) 6.67 Å (c) 3.75 Å (d) 13.3 Å

50. The un-normalized wavefunction of a particle in a spherically symmetric potential is given by $\psi(\bar{r}) = zf(r)$ where f(r) is a function of the radial variable r. The eigenvalue of the operator $\bar{L}^2$ (namely the square of the orbital angular momentum) is

(a) $\hbar^2/4$ (b) $\hbar^2/2$ (c) $\hbar^2$ (d) $2\hbar^2$

51. If ψ_{nlm} denotes the eigenfunction of the Hamiltonian with a potential V = V(r) then the expectation value of the operator $L_x^2 + L_y^2$ in the state $\psi = \frac{1}{5}[3\psi_{211} + \psi_{210} - \sqrt{15}\psi_{21-1}]$ is

(a) $39\hbar^2/25$ (b) $13\hbar^2/25$ (c) $2\hbar^2$ (d) $26\hbar^2/25$

52. If $\psi(x) = A\exp(-x^4)$ is the eigenfunction of a one dimensional Hamiltonian with eigenvalue E = 0, the potential $V(x)$ (in units where $\hbar = 2m = 1$) is

(a) $12x^2$ (b) $16x^6$ (c) $16x^6 + 12x^2$ (d) $16x^6 - 12x^2$

53. An electron in the ground state of the hydrogen atom has the wave function

$$\psi(\vec{r})\frac{1}{\sqrt{\pi a_0^3}}e^{-(r/a_0)}$$

where a_0 is constant. The expectation value of the operator $\hat{Q}=z^2-r^2$, where $z=r\cos\theta$ is (Hint: $\int_0^\infty e^{-ar}r^n dr=\frac{\Gamma(n)}{\alpha^{n+1}}=\frac{(n-1)!}{\alpha^{n+1}}$)

(a) $-\frac{a_0^2}{2}$ (b) $-a_0^2$ (c) $-\frac{3a_0^2}{2}$ (d) $-2a_0^2$

54. A particle of mass 'm' is subjected to a potential,

$$V(x,y)=\frac{1}{2}m\omega^2\left(x^2+y^2\right),\quad -\infty\le x\le\infty,-\infty\le y\le\infty$$

The state with energy $4\hbar\omega$ is g-fold degenerate. The value of g is _____

55. A hydrogen atom is in the state

$$\psi=\sqrt{\frac{8}{21}}\psi_{200}-\sqrt{\frac{3}{7}}\psi_{310}+\sqrt{\frac{4}{21}}\psi_{321}$$

where n,ℓ,m in $\psi_{n\ell m}$ denote the principal, orbital and magnetic quantum numbers, respectively. If $\vec{L}$ is the angular momentum operator, the average value of L^2 is _____ $\hbar^2$.

56. A particle of mass m in three dimensions is in the potential

$$V(r)=\begin{cases}0 & r<a\\ \infty & r\ge a\end{cases}$$

Its ground state energy is

(a) $\frac{\pi^2\hbar^2}{2ma^2}$ (b) $\frac{\pi^2\hbar^2}{ma^2}$ (c) $\frac{3\pi^2\hbar^2}{2ma^2}$ (d) $\frac{9\pi^2\hbar^2}{2ma^2}$

57. Given that $\hat{p}_r=-i\hbar\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)$, the uncertainty Δp_r in the ground state

$$\psi_0(r)=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$$

of the hydrogen atom is

(a) $\frac{\hbar}{a_0}$ (b) $\frac{\sqrt{2}\hbar}{a_0}$ (c) $\frac{\hbar}{2a_0}$ (d) $\frac{2\hbar}{a_0}$

58. A particle of mass m in the potential $V(x,y)=\frac{1}{2}m\omega^2\left(4x^2+y^2\right)$, is in an eigenstate of energy $E=\frac{5}{2}\hbar\omega$. The corresponding un-normalized eigenfunction is

(a) $y\exp\left[-\frac{m\omega}{2\hbar}\left(2x^2+y^2\right)\right]$ (b) $x\exp\left[-\frac{m\omega}{2\hbar}\left(2x^2+y^2\right)\right]$

(c) $y\exp\left[-\frac{m\omega}{2\hbar}\left(x^2+y^2\right)\right]$ (d) $xy\exp\left[-\frac{m\omega}{2\hbar}\left(x^2+y^2\right)\right]$

59. An electron is in the ground state of a hydrogen atom. The probability that it is within the Bohr radius is approximately equal to

(a) 0.60 (b) 0.90 (c) 0.16 (d) 0.32

[ANSWERS]

1. (a)	**2. (c)**	**3. (d)**	**4. (b)**	**5. (a)**	**6. (d)**	**7. (d)**
8. (b)	**9. (d)**	**10. (c)**	**11. (c)**	**12. (c)**	**13. (c)**	**14. (a)**
15. (c)	**16. (c)**	**17. (b)**	**18. (b)**	**19. (d)**	**20. (c)**	**21. (b)**
22. (c)	**23. (b)**	**24. (a)**	**25. (a)**	**26. (c)**	**27. (d)**	**28. (d)**
29. (c)	**30. (a)**	**31. (c)**	**32. (c)**	**33. (c)**	**34. (d)**	**35. (d)**
36. (b)	**37. (c)**	**38. (d)**	**39. (c)**	**40. (c)**	**41. (b)**	**42. (c)**
43. (a)	**44. (a)**	**45. (c)**	**46. (d)**	**47. (c)**	**48. (d)**	**49. (b)**
50. (d)	**51. (d)**	**52. (d)**	**53. (d)**	**54. (3.99 to 4.01)**		
55. (1.99 to 2.01)		**56. (a)**	**57. (a)**	**58. (a)**	**59. (d)**	

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Assignment – 7: Identical Particles

1. A system consists of two identical particles, each of which can be in any one of the two quantum states. The allowed states are

(a) 3 symmetric and antisymmetric (b) 1 symmetric and 3 antisymmetric
(c) 1 symmetric and 1 antisymmetric (d) 3 symmetric and 3 antisymmetric.

2. Consider a one dimensional infinite square well potential $v(x)=\begin{cases}0, \text{ for } 0\le x\le L\\ \infty, \text{ otherwise}\end{cases}$.

If two identical non- interacting bosons occupy the lowest two energy levels, the unnormalized wavefunction of the combined system is given by

(a) $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(b) $\psi(x_1,x_2)=\frac{1}{2}\left[\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)-\sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{2\pi x_1}{L}\right)\right]$

(c) $\psi(x_1,x_2)=\frac{1}{2}\left[\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{2\pi x_1}{L}\right)\right]$

(d) $\psi(x_1,x_2)=\frac{1}{2}\left[\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)\right]$

3. Consider two collision processes one involving two identical $\frac{1}{2}$ fermions and the other involving two identical spin 1 bosons. If in both cases the particles are in relative orbital angular momentum state $l=1$, the spinoral parts of wave functions must be

(a) odd for fermions and even for bosons (b) odd in both cases
(c) even in both cases (d) even for fermions and odd for bosons

4. Consider the wavefunction $\psi=\psi(\vec{r}_1,\vec{r}_2)\chi_s$ for a fermionic system consisting of two spin-half particles. The spatial part of the wavefunction is given by

$$\psi(\vec{r}_1,\vec{r}_2)=\frac{1}{\sqrt{2}}\left[\phi_1(\vec{r}_2)\phi_2(\vec{r}_2)+\phi_2(\vec{r}_1)\phi_1(\vec{r}_2)\right]$$

Where ϕ_1 and ϕ_2 are single particle states. The spin part χ_s of the wavefunction with spin states $\alpha\left(+\frac{1}{2}\right)$ and $\beta\left(-\frac{1}{2}\right)$ should be

(a) $\frac{1}{\sqrt{2}}(\alpha\beta+\beta\alpha)$ (b) $\frac{1}{\sqrt{2}}(\alpha\beta-\beta\alpha)$ (c) $\alpha\alpha$ (d) $\beta\beta$

5. The wavefunctions of two identical particles in state n and s are given by $\phi_n(r_1)$ and $\phi_s(r_2)$, respectively. The particles obey Maxwell-Boltzmann statistics. The state of the combined two-particle system is expressed as

(a) $\phi_n(r_1)+\phi_s(r_2)$ (b) $\frac{1}{\sqrt{2}}\left[\phi_n(r_1)+\phi_s(r_2)+\phi_n\phi_s(r_1)\right]$

(c) $\frac{1}{\sqrt{2}}\left[\phi_n(r_1)+\phi_s(r_2)-\phi_n\phi_s(r_1)\right]$ (d) $\phi_n(r_1)\phi_s(r_2)$

6. The eigenfunction for the Hamiltonian of a particle of mass m in a one dimensional potential V(x) is given by $\psi(x)=A\exp\left(-bx^2/2\right)$, where A and b are constants. It follows that

(a) V(x) = constant (b) $V(x)\propto x^3$ (c) $V(x)\propto x^2$ (d) $V(x)\propto 1/x$

7. A free particle in one dimension is in a state described by

$$\psi(x,t)=A\exp\left((ipx-iEt)/h\right)+B\exp\left((-ipx-iEt)/h\right)$$

The probability current density is given by

(a) p / m (b) $\left(|A|-|B|\right)p/m$ (c) $\left(|A|^2-|B|^2\right)p/m$ (d) $\left(|A|^2+|B|^2\right)p/m$

8. Consider a one-dimensional infinite square well potential

$$V(x)=\begin{cases}0 & \le x\le L\\ \infty & \text{otherwise}\end{cases}$$

If each of the lowest two energy levels are occupied by identical non-interacting bosonic particles (one in each level), then the unnormalized wave function of the combined system is:

(a) $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(b) $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{2\pi x_1}{L}\right)$

(c) $\psi(x_1,x_2)=\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(d) $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)+\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

9. Lithium has three electrons in the state $\psi_1(\vec{r}_1);\psi_2(\vec{r}_2)$ and $\psi_3(\vec{r}_3)$. The wavefunction of the three electrons (together) can be written as

(a) $\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\psi_3(\vec{r}_3)$ (b) $\left(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)-\psi_1(\vec{r}_1)\psi_2(\vec{r}_1)\psi_3(\vec{r}_3)\right)$

(c) $\left(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)+\psi_1(\vec{r}_1)\psi_2(\vec{r}_1)\psi_3(\vec{r}_3)\right)$ (d) $\frac{1}{\sqrt{6}}\begin{vmatrix}\psi_1(\vec{r}_1) & \psi_1(\vec{r}_2) & \psi_1(\vec{r}_3)\\ \psi_2(\vec{r}_1) & \psi_2(\vec{r}_2) & \psi_3(\vec{r}_3)\\ \psi_3(\vec{r}_1) & \psi_3(\vec{r}_2) & \psi_3(\vec{r}_3)\end{vmatrix}$

10. Two electrons are confined in a one dimensional box of length L. The one-electron states are given by $\psi_n(x) = \sqrt{2/L}\sin(n\pi x/L)$. What would be the ground state wave function $\psi(x_1, x_2)$ if both electrons are arranged to have the same spin state?

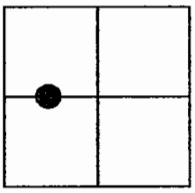
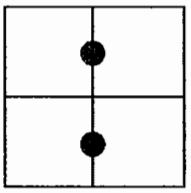
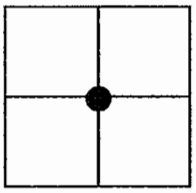
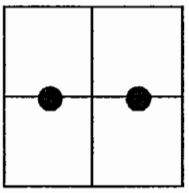
(a) $\psi(x_1,x_2) = \frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) + \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

(b) $\psi(x_1,x_2) = \frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) - \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

(c) $\psi(x_1,x_2) = \frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(d) $\psi(x_1,x_2) = \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)$

11. In a Stern-Gerlach experiment with spin-½ particles, the beam is found to form two spots on the screen, one directly above the other. The experimenter now makes a hole in the screen at the position of the upper spot. The particles that go through this hole are then passed through another Stern-Gerlach apparatus but with its magnets rotated by 90 degrees counter-clockwise about the axis of the beam direction. Which of the following shows what happens on the second screen?

(a)  (b)  (c)  (d) 

12. The ground state and the first excited state wave functions of a one dimensional infinite potential well are ψ_1 and ψ_2, respectively. When two spin-up electrons are placed in this potential, which one of the following, with x_1 and x_2 denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system?

(a) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_1) - \psi_1(x_2)\psi_2(x_2)]$ (b) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)]$

(c) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_1) + \psi_1(x_2)\psi_2(x_2)]$ (d) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)]$

13. Which one of the following is a fermion?

(a) α – particle (b) ${}_4Be^7$ nucleus (c) Hydrogen atom (d) Deuteron

14. If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular momentum, then $\vec{L}.\vec{S}$ does not commute with

(a) S_z (b) L^2 (c) S^2 (d) $(\vec{L}+\vec{S})^2$

15. Consider a system of two non-interacting identical fermions, each of mass m in an infinite square well potential of width a. (Take the potential inside the well to be zero and ignore spin). The composite wavefunction for the system with total energy

$$E = \frac{5\pi^2\hbar^2}{2ma^2}$$

is

(a) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) - \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(b) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) + \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(c) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{2a}\right) - \sin\left(\frac{3\pi x_1}{2a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(d) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\cos\left(\frac{\pi x_2}{a}\right) - \sin\left(\frac{\pi x_2}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

[ANSWERS]

1. (a)	**2. (c)**	**3. (a)**	**4. (b)**	**5. (d)**	**6. (c)**	**7. (c)**
8. (b)	**9. (d)**	**10. ( )**	**11. (d)**	**12. (b)**	**13. (b)**	**14. (a)**
15. (a)						

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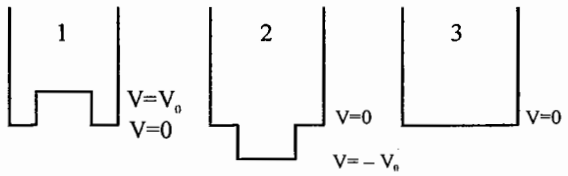
Assignment – 8: Pertubation Theory

1. A quantum harmonic oscillator is in the energy eigenstate $|n\rangle$. A time independent perturbation $\lambda(a^\dagger a)^2$ acts on the particle, where λ is a constant of suitable dimensions and a and $a^\dagger$ are lowering and raising operators respectively. Then the first order energy shift is gien by

(a) λn (b) $\lambda^2 n$ (c) λn^2 (d) $(\lambda n)^2$

2. Let E_1, E_2, E_3 be the respective ground state energies of the following potentials: Which one of the following is correct?



(a) $E_1 < E_2 < E_3$ (b) $E_3 < E_1 < E_2$ (c) $E_2 < E_3 < E_1$ (d) $E_2 < E_1 < E_3$.

3. The validity of Fermi's golden rule which connects two states with energy difference ΔE, implies that it is true for

(a) All times t (b) $t \geq \frac{\hbar}{\Delta E}$ (c) $t \leq \frac{\hbar}{\Delta E}$ (d) $\Delta E = 0$

4. If the perturbation $H' = ax$, where a is a constant; is added to the infinite square well potential

$$V(x) = \begin{cases} 0 \text{ for } & 0 \leq x \leq \pi \\ \infty & \text{otherwise} \end{cases}$$

The first order correction to ground state energy is:

(a) $\frac{a\pi}{2}$ (b) $a\pi$ (c) $\frac{a\pi}{4}$ (d) $\frac{a\pi}{\sqrt{2}}$

5. The perturbation $H' = bx^2$, where 'b' is a constant, is added to the one dimensional harmonic oscillator potential $V(x) = \frac{1}{2}m\omega^2 x^2$. Which of the following denotes the correction to the ground state energy to first order in b ?

[Hint : The normalized ground state wave function of the one dimensional harmonic oscillator potential is $\psi_0 = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} e^{-m\omega x^2/2\hbar}$. You may use the following integral $\int_{-\infty}^{\infty} x^{2n} e^{-ax^2} dx = a^{-n-\frac{1}{2}}\Gamma\left(n+\frac{1}{2}\right)$

(a) $\frac{3b\hbar^2}{4m^2\omega^2}$ (b) $\frac{3b\hbar^2}{2m^2\omega^2}$ (c) $\frac{3b\hbar^2}{2\pi m^2\omega^2}$ (d) $\frac{15b\hbar^2}{4m^2\omega^2}$

6. A constant perturbation as shown in the figure below acts on a particle of mass m confined in a infinite potential well between 0 and L.

The first-order correction to the ground state energy of the particle is

(a) $\frac{V_0}{2}$ (b) $\frac{3V_0}{4}$

(c) $\frac{V_0}{4}$ (d) $\frac{3V_0}{2}$

7. A one-dimensional harmonic oscillator whose Hamiltonian is $H_0 = \frac{1}{2m}\hat{p}^2 + \frac{1}{2m}\omega^2\hat{x}^2$ is perturbed such that the total Hamiltonian is $H = H_0 + \varepsilon\hat{x}$. The first order correction to the ground state is

(a) $2\varepsilon\hbar\omega$ (b) $\varepsilon\hbar\omega/2$ (c) $\varepsilon\hbar\omega$ (d) 0

8. The first-order correction to the eigenvalues of the Hamiltonian $\hat{H}_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2$ due to a perturabation $\hat{H}_1 = \alpha\hat{x}$ are

(a) 0 (b) α/k (c) $2\alpha/k$ (d) $-2\alpha/k$

9. Consider a system in the unperturbed state described by the Hamiltonian, $H_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. The system is subjected to a perturbation of the form $H' = \begin{pmatrix} \delta & \delta \\ \delta & \delta \end{pmatrix}$, where $\delta << 1$. The energy eigenvalues of the perturbed system using the first order perturbation approximation are

(a) 1 and $(1+2\delta)$ (b) $(1+\delta)$ and $(1-\delta)$ (c) $(1+2\delta)$ and $(1-2\delta)$ (d) $(1+\delta)$ and $(1-2\delta)$

Common data for Q.10, Q.11 and Q. 12.

An unperturbed two-level system has energy eigenvalues E1 and E2, and eigen functions $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

When perturbed, its Hamiltonian is represented by $\begin{pmatrix} E_1 & A \\ A^* & E_2 \end{pmatrix}$

10. The first-order correction to E_1 is:

(a) 4A (b) 2A (c) A (d) 0

11. The second-order correction to E_1 is

(a) 0 (b) A (c) $\frac{A^2}{E_2 - E_1}$ (d) $\frac{A^2}{E_1 - E_2}$

12. The first-order correction to the eigenfunction $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is:

(a) $\begin{pmatrix} 0 \\ A^*/(E_1 - E_2) \end{pmatrix}$ (b) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (c) $\begin{pmatrix} A^*/(E_1 - E_2) \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

13. The wave function of a one-dimensional harmonic oscillator is $\psi_0 = A\exp\left(\frac{-\alpha^2x^2}{2}\right)$ for the ground state E_0 $(\alpha x/10)^4$, the first order change in the ground state energy is: [Given: $\left[(x+1) = \int_0^\infty t^x \exp(-t)dt\right]$

(a) $\left(\frac{1}{2}E_0\right)10^{-4}$ (b) $(3E_0)10^{-4}$ (c) $(4E_0)10^{-4}$ (d) $(E_0)10^{-4}$

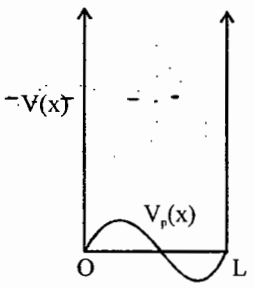
14. Let E'_n $(n = 0,1,2,....)$ be the energy eigenvalues for a particle of mass m placed in an anharmonic potential $V(x) = \frac{1}{2}m\omega^2x^2 + ax^4, (a > 0)$. Let $E_n \equiv \left(n+\frac{1}{2}\right)\hbar\omega$. Then according to the first order perturbation theory:

(a) $E'_0 = E_0$ (b) $E'_0 > E_0$ (c) $E'_0 < E_0$ (d) $E'_0 < E_0$ for all n

15. A particle of mass m is confined in an infinite potential well:

$$V(x) = \begin{cases} 0 & \text{if } 0 < x < L, \\ \infty & \text{otherwise} \end{cases}$$

It is subjected to a perturbing potential $V_p(x) = V_0 \sin\left(\frac{2\pi x}{L}\right)$ within the well. Let $E^{(1)}$ and $E^{(2)}$ be the corrections to the ground state energy in the first and second order in V_0, respectively. Which of the following are true?



(a) $E^{(1)} = 0;\ E^{(2)} < 0$ (b) $E^{(1)} > 0;\ E^{(2)} = 0$

(c) $E^{(1)} = 0;\ E^{(2)}$ depends on the sign of V_0 (d) $E^{(1)} < 0;\ E^{(2)} < 0$

16. Consider an electron in a box of lenght L with periodic boundary condition $\psi(x) = \psi(x+L)$. If the electron is in the $\psi_k(x) = \frac{1}{\sqrt{L}}e^{jkx}$ with energy $\varepsilon_k = \frac{\hbar^2k^2}{2m}$, what is the correction to its energy, to second order of perturbation theory, when it is subjected to weak periodic potential $V(x) = V_0 \cos gx$, where g is an integral multiple of the $2\pi/L$?

(a) $V_0^2\,\varepsilon_g/\varepsilon_k^2$ (b) $-\frac{mV_0^2}{2\hbar^2}\left(\frac{1}{g^2+2kg}+\frac{1}{g^2-2kg}\right)$

(c) $V_0^2(\varepsilon_k - \varepsilon_g)/\varepsilon_g^2$ (d) $V_0^2/(\varepsilon_k + \varepsilon_g)$

17. The energy eigenvalues of a particle in the potential $V(x) = \frac{1}{2}m\omega^2x^2 - ax$ are

(a) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega - \frac{a^2}{2m\omega^2}$ (b) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega + \frac{a^2}{2m\omega^2}$

(c) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega - \frac{a^2}{m\omega^2}$ (d) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$

18. The perturbation

$$H' = \begin{cases} b(a-x) & -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

acts on a particle of mass 'm' confined in an infinite square well potential

$$V(x) = \begin{cases} 0 & -a < x < a \\ \infty & \text{otherwise} \end{cases}$$

The first order correction to the ground state energy of the particle is

(a) $\frac{ba}{2}$ (b) $\frac{ba}{\sqrt{2}}$ (c) 2ba (d) ba

Common Data for Q. 19 and Q. 20:

To the given unperturbed Hamiltonian $\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

we add a small perturbation given by $\varepsilon\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

where ε is a small quantity.

19. The ground state eigen vector of the unperturbed Hamiltonian is

(a) $(1/\sqrt{2}, 1/\sqrt{2}, 0)$ (b) $(1/\sqrt{2}, -1/\sqrt{2}, 0)$ (c) (0, 0, 1) (d) (1, 0, 0)

20. A pair of eigen values of the perturbed Hamiltonian, using first order perturbation theory, is

(a) $3+2\varepsilon, 7+2\varepsilon$ (b) $3+2\varepsilon, 2+\varepsilon$ (c) $3, 7+2\varepsilon$ (d) $3, 2+2\varepsilon$

21. Consider a two-dimensional infinite square will $V(x,y) = \begin{cases} 0 & 0 < x < a, \quad 0 < y < a \\ \infty & \text{otherwise} \end{cases}$. Its normalized eigenfunctions are $\psi_{n_x, n_y}(x,y) = \frac{2}{a}\sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{a}\right)$ where n_x, n_y = 1, 2, 3, ... If a perturbation $H' = \begin{cases} V_0 & 0 < x < \frac{a}{2}, \quad 0 < y < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$ is applied, then the correction to the energy of the first excited state to order V_0 is

(a) $\frac{V_0}{4}$ (b) $\frac{V_0}{4}\left[1 \pm \frac{64}{9\pi^2}\right]$ (c) $\frac{V_0}{4}\left[1 \pm \frac{16}{9\pi^2}\right]$ (d) $\frac{V_0}{4}\left[1 \pm \frac{32}{9\pi^2}\right]$

22. A particle of mass m is contained in a one-dimensional infinite well extending from $x = -L/2$ to $x = L/2$. The particle is in its ground state given by $\psi_0(x) = \sqrt{2/L}\cos(\pi x / L)$. The walls of the box are moved suddenly to form a box extending from $x = -L$ to $x = L$. What is the probability that the particle will be in the ground state after this sudden expansion?

(a) $(8/3\pi)^2$ (b) 0 (c) $(16/3\pi)^2$ (d) $(4/3\pi)^2$

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Assignment – 9: Scattering Theory

1. If σ is the total cross-section and $f(\theta)$, θ being the angle of scattering is the scattering amplitude for a quantum mechanical elastic scattering by a spherically symmetric potential, then which of the following is true? Note that k is the magnitude of the wave vector along the $\hat{z}$ direction.

(a) $\sigma = |f(\theta)|^2$ (b) $\sigma = \frac{4\pi}{k}|f(\theta=0)|^2$

(c) $\sigma = \frac{4\pi}{k} \times$ imaginary part of $[f(\theta=0)]$ (d) $\sigma = \frac{4\pi}{k}|f(\theta)|^2$

2. For the scattering under a central force field, the asymptotic form of the total wave function after scattering is:

(a) $\psi(r) = e^{ikr} + e^{-ikr}$ (b) $\psi(r) = f(0)\frac{e^{-ik.r}}{r}$

(c) $\psi(r) = e^{ik.r} + f(0)e^{ik.r}$. (d) $\psi(r) = e^{ik.r} + f(\theta)\frac{e^{ik.r}}{r}$

3. Born approximation is justified for
(a) Particle moving very slowly. (b) Very strong potential fields.
(c) Fast particles in a weak potential. (d) Hydrogen atom.

4. A beam of quantum mechanical particles of momentum k moving along the z-direction scatters off a spherically symmetric potential

$$V(r) = \begin{cases} V_0, & \text{for } 0 \le r \le R \\ 0, & \text{for } r > R \end{cases}$$

The leading term of the wave function of the scattered particles is of the form

(a) e^{ikr} (b) e^{ikz} (c) $\frac{e^{ikr}}{r}$ (d) $\frac{e^{ikr}}{r^2}$

Common data for Q. 5 and Q. 6.

A particle is scattered by a spherically symmetric potential. In the centre of mass (CM) frame the wavefunction of the incoming particle is $\psi = Ae^{ikz}$ where k is the wavevector and A is a constant.

5. If $f(\theta)$ is an angular function then in the asymptotic region the scattered wavefunction has the form

(a) $\frac{Af(\theta)e^{ikr}}{r}$ (b) $\frac{Af(\theta)e^{-ikr}}{r}$ (c) $\frac{Af(\theta)e^{ikr}}{r^2}$ (d) $\frac{Af(\theta)e^{-ikr}}{r^2}$

6. The differential scattering cross section $\sigma(0)$ in CM frame is:

(a) $\sigma(\theta) = |A|^2\frac{|f(\theta)|^2}{r^2}$ (b) $\sigma(\theta) = |A|^2|f(\theta)|^2$

(c) $\sigma(\theta) = |f(\theta)|^2$ (d) $\sigma(\theta) = |A||f(\theta)|$

7. Let k be the wave number of the incident plane wave in a scattering experiment. If the scattering is purely a p-wave with the phase shift $\delta_1 = \pi/4$, then the total scattering cross-section is:

(a) $2\pi/k^2$ (b) $6\pi/k^2$ (c) 0 (d) k^2

8. For a spherically symmetric potential, $V(r) = g\delta(\vec{r})$ where $\delta(\vec{r})$ is a three dimensional δ – function and g is a constant. The scattering amplitude f(0) in the Born approximation then varies as

(a) g (b) g^2 (c) 1/g (d) $1/g^2$.

9. A beam of particles of linear momentum $\hbar k$ is scattered by a short-range potential of range a_1 where $ak << 1$. Which of the following statements is correct?

(a) All phase shifts are zero (b) There is no scattering
(c) The scattering is spherically symmetric (d) All phase shifts are equal.

10. Consider the scattering of two identical fermions. If the two particles are in a spin singlet state, the differential cross section relates to the scattering amplitude as

(a) $\frac{d\sigma}{d\Omega} = |f(\theta)|^2 + |f(\pi-\theta)|^2$ (b) $\frac{d\sigma}{d\Omega} = |f(\theta)|^2 - |f(\pi-\theta)|^2$

(c) $\frac{d\sigma}{d\Omega} = |f(\theta) - f(\pi-\theta)|^2$ (d) $\frac{d\sigma}{d\Omega} = |f(\theta) + f(\pi-\theta)|^2$

11. A free particle described by a plane wave and moving in the positive z-direction undergoes scattering by a potential $V(r) = \begin{cases} V_0 & \text{if } r \le R \\ 0 & \text{if } r > R \end{cases}$ if V_0 is changes to $2V_0$, keeping R fixed, then the differential scattering cross-section, in the born approximation,

(a) increases to four times the original value (b) increases to twice the original value
(c) decreases to half the original value (d) decreases to one fourth the original value

12. In the Born approximation, the scattering amplitude $f(\theta)$ for the Yukawa potential $V(r) = \frac{\beta e^{-\mu r}}{r}$ is given by: (in the following $b = 2k \sin\frac{\theta}{2}$, $E = \hbar^2 k^2 / 2m$)

(a) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^2}$ (b) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)}$ (c) $-\frac{2m\beta}{\hbar^2\sqrt{\mu^2+b^2}}$ (d) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^3}$

[ANSWERS]

1. (c) 2. (d) 3. (c) 4. (c) 5. (a) 6. (c) 7. (b)
8. (a) 9. (c) 10. (a) 11. (a) 12. (b)

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Assignment – 10: Miscellaneous

1. Let E_T be the energy obtained variationally by using trial function $Ae^{-\alpha r^2}$ (with α as a variational parameter) for Hydrogen atom. If the exact ground state is E_H then,

   (a) $E_T = E_H$ (b) $E_T > E_H$ (c) $E_T \geq E_H$ (d) $E_T < E_H$

2. The dispersion relation for travelling waves admitted by the wave equation $\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial x^2} + \alpha\frac{\partial^4\psi}{\partial x^4} = 0$ (α : parameter) is:

   (a) $\omega = \pm\sqrt{k^2 + \alpha k^4}$ (b) $\omega = \pm\sqrt{k + \alpha k^2}$

   (c) $\omega = \pm\left(k^2 + \alpha k^4\right)$ (d) $\omega = \pm\left(k^2 + \alpha k^2\right)$

3. Given the Hamiltonian $\hat{H} = \hat{H}_0 + i\hat{T}$ where $\hat{H}_0$ and $\hat{T}$ are hermitian, which of the following statements are correct?

   (a) $\hat{H}$ is a hermitian operator and the total probability is consrved.

   (b) $\hat{H}$ is a hermitian and the total probability is not conserved.

   (c) $\hat{H}$ is not a hermitian operator and the total probability is conserved.

   (d) $\hat{H}$ is not a hermitian operator and the total probability is not conserved.

4. A particle in one dimension moves under the influence of a potential $V(x) = ax^6$, where a is a real constant. For large n the quantized energy level E_n depends on n as:

   (a) $E_n \sim n^3$ (b) $E_n \sim n^{4/3}$ (c) $E_n \sim n^{6/5}$ (d) $E_n \sim n^{3/2}$

5. A variational calculation is done with the normalized trial wavefunction $\psi(x) = \frac{\sqrt{15}}{4a^{5/2}}\left(a^2 - x^2\right)$ for the one dimensional potential well $V(x) = \begin{cases} 0 & \text{if } |x| \leq a \\ \infty & \text{if } |x| > a \end{cases}$

   The ground state energy is estimated to be

   (a) $\frac{5\hbar^3}{3ma^2}$ (b) $\frac{3\hbar^2}{2ma^2}$ (c) $\frac{3\hbar^2}{5ma^2}$ (d) $\frac{5\hbar^2}{4ma^2}$

6. For a harmonic oscillator of mass $m = \frac{1}{2}$ and energy E, moving in a potential $V(x) = \frac{1}{2}kx^2$, the WKB quantization condition reads

   (a) $\int_{-\sqrt{2E/k}}^{\infty} dx\sqrt{E - \frac{1}{2}kx^2} = \left(n + \frac{1}{2}\right)\pi\hbar$ (b) $\int_{\infty}^{\sqrt{2E/k}} dx\sqrt{E - \frac{1}{2}kx^2} = \left(n + \frac{1}{2}\right)\pi\hbar$

   (c) $\int_{-\sqrt{2E/k}}^{\sqrt{2E/k}} dx\sqrt{E - \frac{1}{2}kx^2} = \left(n + \frac{1}{2}\right)\pi\hbar$ (d) $\int_{-\infty}^{\infty} dx\sqrt{E - \frac{1}{2}kx^2} = \left(n + \frac{1}{2}\right)\pi\hbar$

7. What would be the ground state energy of the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

if vibrational principle is used to estimate it with the trial wavefunction $\psi(x) = A\, e^{-bx^2}$ with b as the variational parameter?

[Hint: $\int_{-\infty}^{\infty} x^{2n} e^{-2bx^2} dx = (2b)^{-n-\frac{1}{2}} \Gamma\left(n+\frac{1}{2}\right)$]

(a) $-m\alpha^2/2\hbar^2$ (b) $-2m\alpha^2/\pi\hbar^2$ (c) $-m\alpha^2/\pi\hbar^2$ (d) $m\alpha^2/\pi\hbar^2$

8. The bound on the ground state energy of the Hamiltonian with an attractive delta-function potential, namely

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

using the variational principle with the trial wavefunction $\psi(x) = A\exp(-bx^2)$ is

[Note: $\int_0^{\infty} e^{-t} t^a dt = \Gamma(a+1)$]

(a) $-m\alpha^2/4\pi\hbar^2$ (b) $-m\alpha^2/2\pi\hbar^2$ (c) $-m\alpha^2/\pi\hbar^2$ (d) $-m\alpha^2/\sqrt{5}\pi\hbar^2$

9. A spin $-\frac{1}{2}$ particle is in the state $\chi = \frac{1}{\sqrt{11}}\begin{pmatrix} 1+i \\ 3 \end{pmatrix}$ in the eigenbasis of S^2 and S_z. If we measure S_z the probabilities of getting $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$, respectively, are

(a) $\frac{1}{2}$ and $\frac{1}{2}$ (b) $\frac{2}{11}$ and $\frac{9}{11}$ (c) 0 and 1 (d) $\frac{1}{11}$ and $\frac{3}{11}$

10. Let ψ_{nlm} denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential $V(r)$. The expectation value of L_z in the state

$\psi = \frac{1}{6}\left[\psi_{200} + \sqrt{5}\,\psi_{210} + \sqrt{10}\,\psi_{21-1} + \sqrt{20}\,\psi_{211}\right]$ is

(a) $-\frac{5}{18}\hbar$ (b) $\frac{5}{6}\hbar$ (c) $\hbar$ (d) $\frac{5}{18}\hbar$

11. The probability function for a variable x which assumes only positive values is

$$f(x) = x\exp\left(-\frac{x}{\lambda}\right)$$

where $\lambda > 0$. The ratio $\langle x\rangle / \hat{x}$, where $\hat{x}$ is the most probable value and $\langle x\rangle$ is the mean value of the variable x, is

(a) 2 (b) $\frac{1+\lambda}{1-\lambda}$ (c) $\frac{1}{\lambda}$ (d) 1

12. A particle moving in a one dimension has the un-normalised wave function

$$\psi(x) = x\exp\left(-\frac{x^2}{\lambda^2}\right)$$

where λ is a real constant.The expectation value of its momentum is $\langle p \rangle =$

(a) $\frac{\hbar}{\lambda}\exp\left(-\frac{x^2}{\lambda^2}\right)$ (b) $\frac{\hbar^2}{\lambda^2} - 2\frac{\hbar}{\lambda}$ (c) $\frac{\hbar}{\lambda}\exp(-1)$ (d) zero

13. A particle of mass 'm' and charge 'e' is in the ground state of a one-dimensional harmonic oscillator potential in the presence of a uniform external electric field E. The total potential felt by the particle is

$$V(x) = \frac{1}{2}kx^2 - eEx$$

If the electric field is suddenly switched off, then the particle will

(a) Make a transition to any harmonic oscillator state with $x = -eE/k$ as origin without emitting any photon.

(b) Make a transition to any harmonic oscillator state with $x = 0$ as origin and absorb a photon.

(c) Settle into the harmonic oscillator ground state with $x = 0$ as origin after absorbing a photon.

(d) Oscillate back and forth with initial amplitude eE/k, emitting multiple photons as it does so.

14. Consider the Hamiltonian

$$H = f\vec{\sigma}.\vec{x}$$

Here $\vec{x}$ is the position vector, f is a constant and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, where $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices. The energy eigenvalues are

(a) $f\left(\sqrt{x^2+y^2} \pm z\right)$ (b) $f(x \pm iy)$ (c) $\pm f\sqrt{x^2+y^2+z^2}$ (d) $\pm f(x+y+z)$

15. A rigid rotator is in a quantum state described by the wavefunction

$$\psi(\theta, \varphi) = \sqrt{\frac{3}{4\pi}}\sin\theta\sin\varphi$$

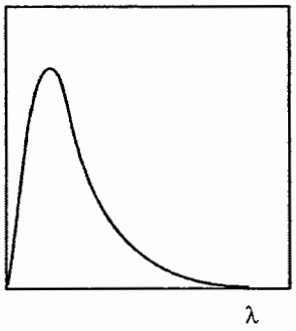
where θ and φ are the usual polar angles. If two successive measurements of L_z are made on this rotator, the probability that the second measurement will yield the value $+\hbar$ is

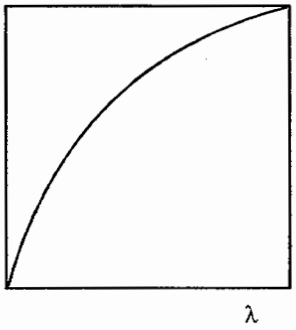
(a) 0.25 (b) 0.33 (c) 0.5 (d) negligible

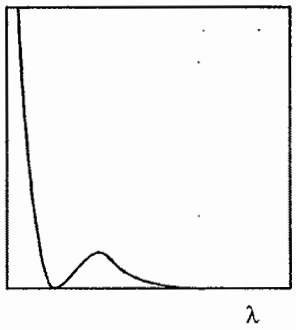
16. A particle in the 2s state of hydrogen has the wave function

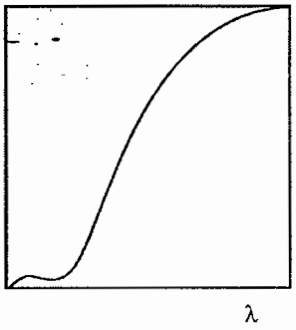
$$\psi_{2s}(r)=\frac{1}{4\sqrt{2}\,\pi}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{r}{a_0}\right)\exp\left(-\frac{r}{2a_0}\right)$$

where 'r' is the radial coordinate w.r.t. the nucleus as origin and a_0 is the Bohr radius. The probability P of finding the electron somewhere inside a sphere of radius λa_0 centred at the nucleus, is best described by the graph

(a) 

(b) 

(c) 

(d) 

17. Which one of the following is not true for a hydrogen atom?
(a) The components of angular momentum do not commute with each other
(b) The components of angular momentum commute with the total angular momentum
(c) The total energy is independent of principle quantum number n
(d) The energy level corresponding to the principal quantum number n has n^2 fold degeneracy

18. If a proton were ten times lighter, the ground state energy of the electron in a hydrogen atom would be
(a) less (b) more (c) the same
(d) less, more or equal depending on the electron mass

19. The recoil momentum of an atom is pA when it emits an infrared photon of wavelength 1500 nm, and it is pB when it emits a photon of visible wavelength 500 nm. The ratio $\frac{p_A}{p_B}$ is:

(a) 1 : 1 (b) $1:\sqrt{3}$ (c) 1 : 3 (d) 3 : 2

Assignment -10: Miscellaneous

[ANSWERS]

1. (c)	**2. (a)**	**3. (d)**	**4. (a)**	**5. (d)**	**6. (c)**	**7. (c)**
8. (c)	**9. ( )**	**10. ( )**	**11. (a)**	**12. (d)**	**13. (b)**	**14. (c)**
15. (c)	**16. (d)**	**17. (b)**	**18. ( )**	**19. (c)**		